

Backpropagation

CSCI 601 471/671
NLP: Self-Supervised Models

<https://self-supervised.cs.jhu.edu/sp2023/>



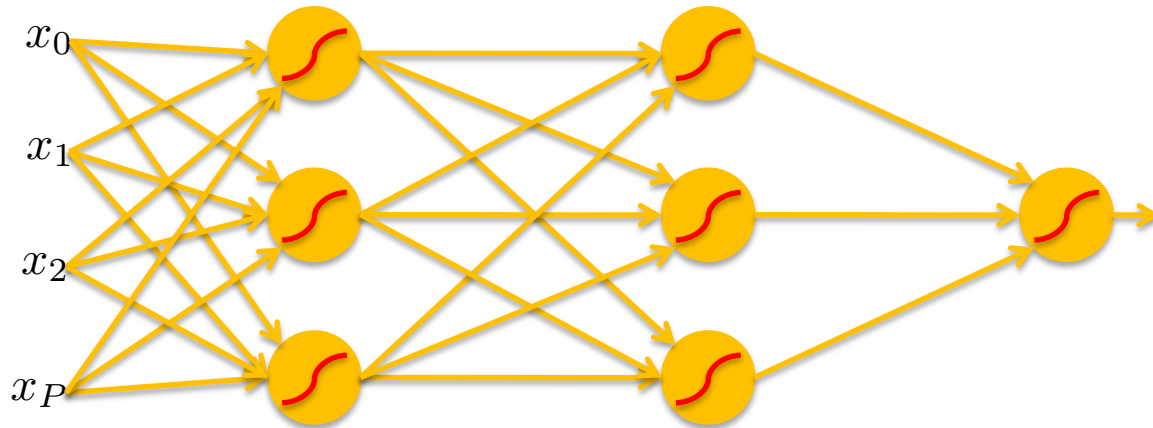
JOHNS HOPKINS
UNIVERSITY

[Slide credit: Andrej Karpathy and many others]

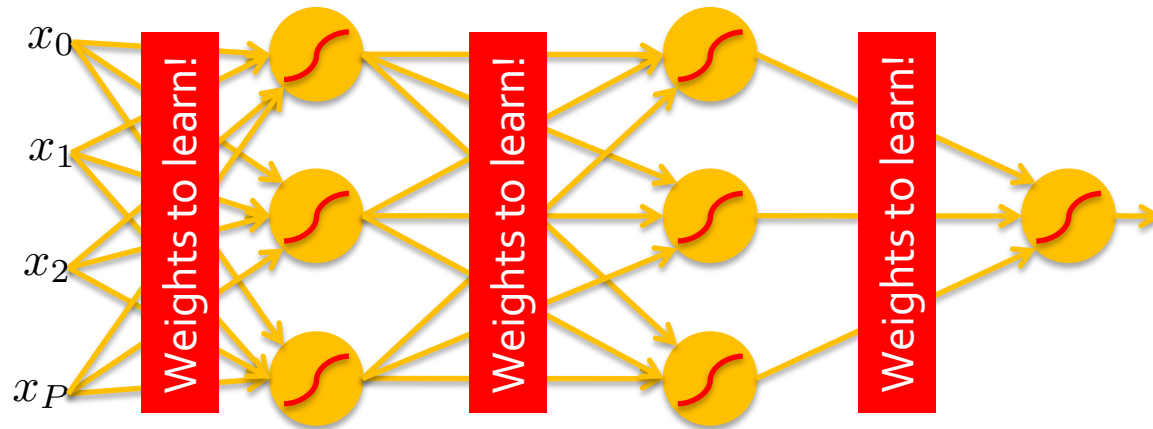
HW update

- HW1 grades are up!
 - Stats: Mean: 93.1 (std: ~5)
 - There was a mistake in grading Q4.6, but should be corrected now.
- Regrade requests can be submitted via Gradescope.
 - Please don't spam us! 🙏
- HW3 is up!
 - Focus: training neural networks

Recap: Feed Forward Neural Networks



Recap: Feed Forward Neural Networks



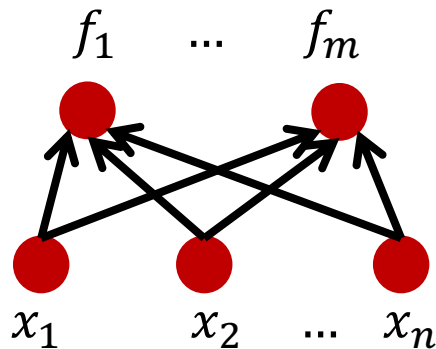
Recap: Jacobian Matrix

- Generalization of gradients
- Given a function with **m outputs** and **n inputs**

$$\mathbf{f}(\mathbf{x}) = [f_1(x_1, x_2, \dots, x_n), \dots, f_m(x_1, x_2, \dots, x_n)] \in \mathbb{R}^m$$

- It's Jacobian is an **$m \times n$ matrix** of partial derivatives:

$$\mathbf{J}_f(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \in \mathbb{R}^{m \times n}$$



Recap: Chain Rule for Multivariable Functions

- Looks similar to the single-variable setup:

$$\mathbf{J}_{\mathbf{f} \circ \mathbf{g}}(\mathbf{x}) = \mathbf{J}_{\mathbf{f}}(\mathbf{g}(\mathbf{x})) \mathbf{J}_{\mathbf{g}}(\mathbf{x})$$

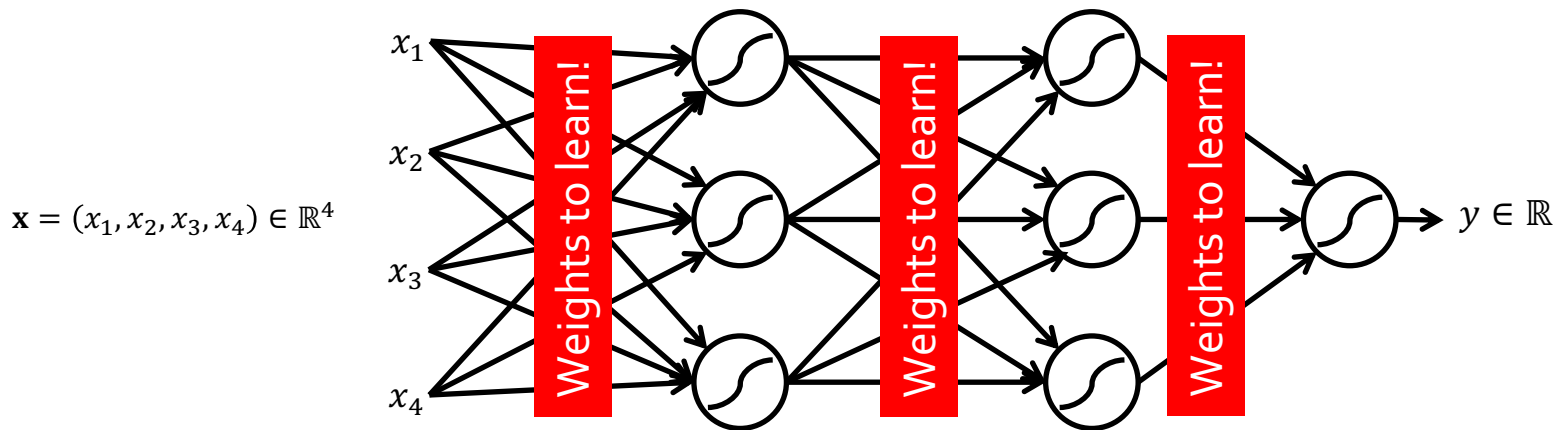


Note, the above statement is a **matrix** multiplication!

Function $\mathbf{f} \circ \mathbf{g}$ has m outputs and d inputs $\rightarrow m$ by d Jacobian

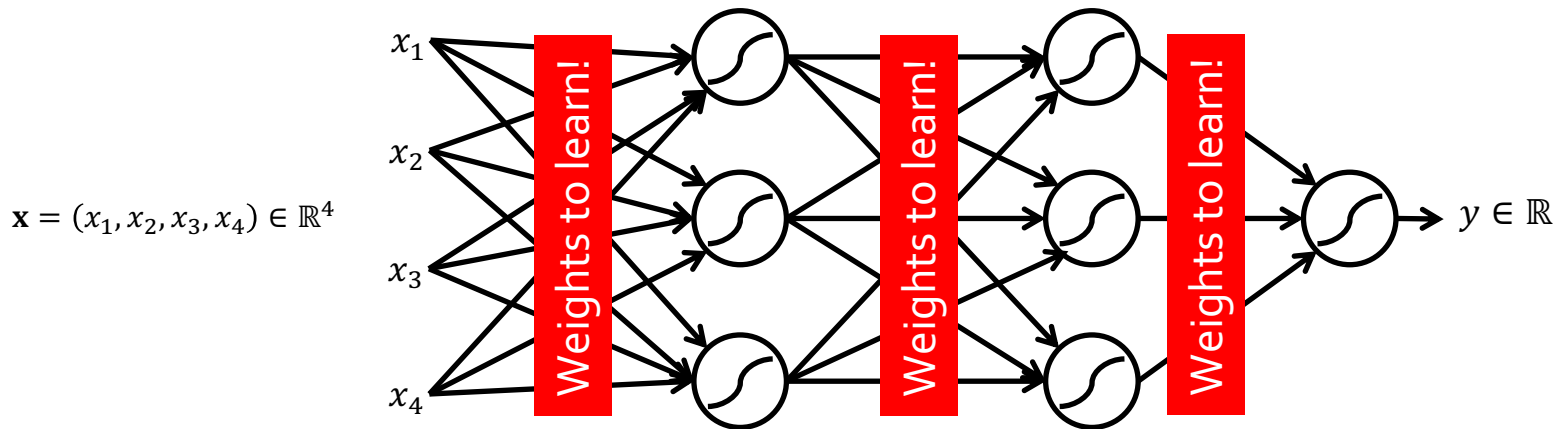
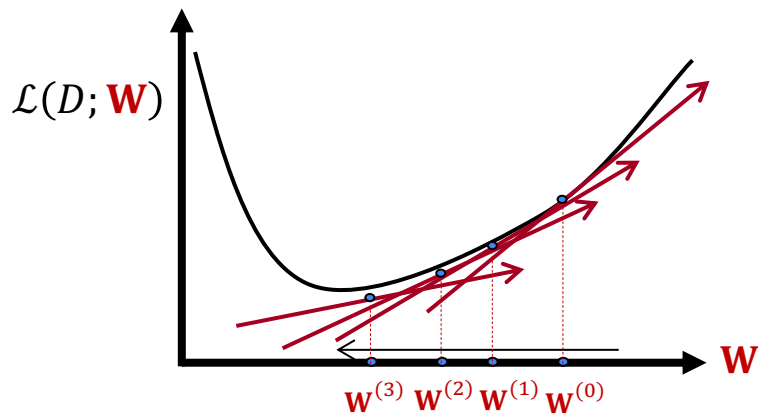
Training Neural Networks: Setup

- We are given an architecture though its weights \mathbf{W} .
- We are given a loss function $\ell: \mathbb{R} \times \mathbb{R} \rightarrow (0, 1)$
 - $\ell(y^*, y)$ quantifies distance between an answer y^* and prediction $y = \text{NN}(\mathbf{x}; \mathbf{W})$ — lower is better
- Also given a training data $D = \{(\mathbf{x}_i, y_i^*)\}$
- Overall objective to optimize: $\mathcal{L}(D; \mathbf{W}) = \sum_{(\mathbf{x}_i, y_i^*) \in D} \ell(\text{NN}(\mathbf{x}_i; \mathbf{W}), y_i^*)$



Training Neural Networks ~ Optimizing Parameters

- We can use **gradient descent** to minimize the loss.
- At each step, the **weight vector** is modified in the **direction that produces the steepest descent** along the error surface.

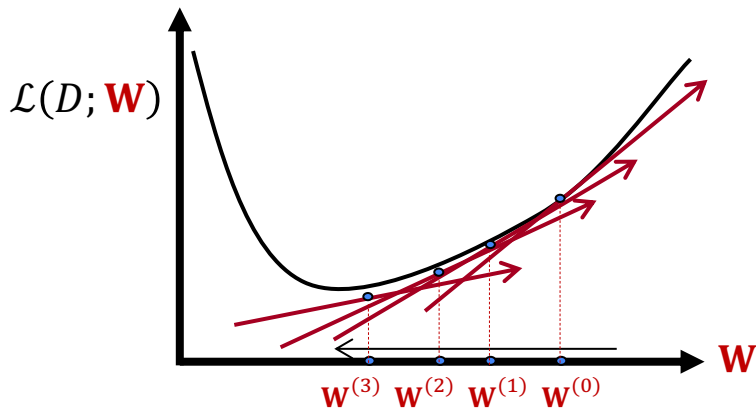


Training Neural Networks ~ Optimizing Parameters

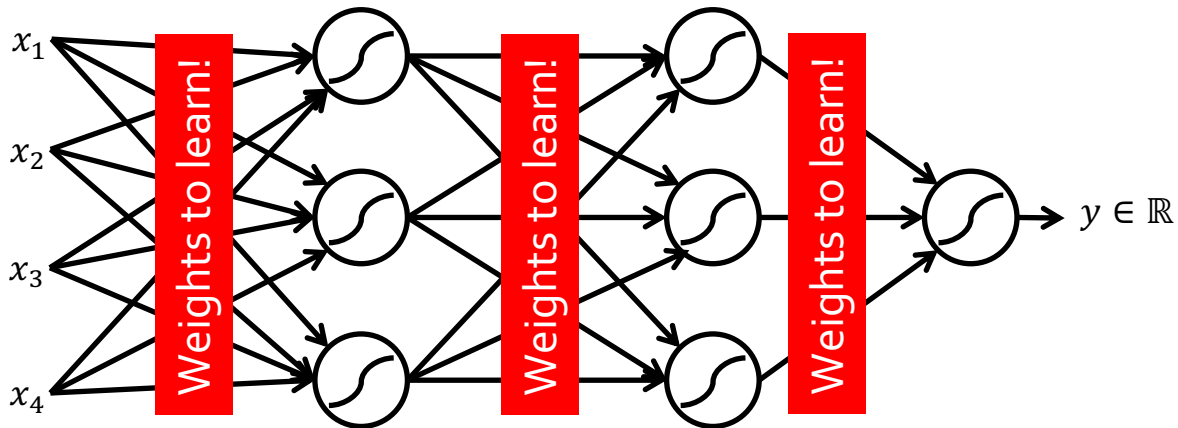
For each sub-parameter $W_i \in \mathbf{W}$:

$$W_i^{(t+1)} = W_i^{(t)} - \alpha \frac{\partial \mathcal{L}}{\partial W_i}$$

It all comes down to effectively computing $\frac{\partial \mathcal{L}}{\partial W_i}$

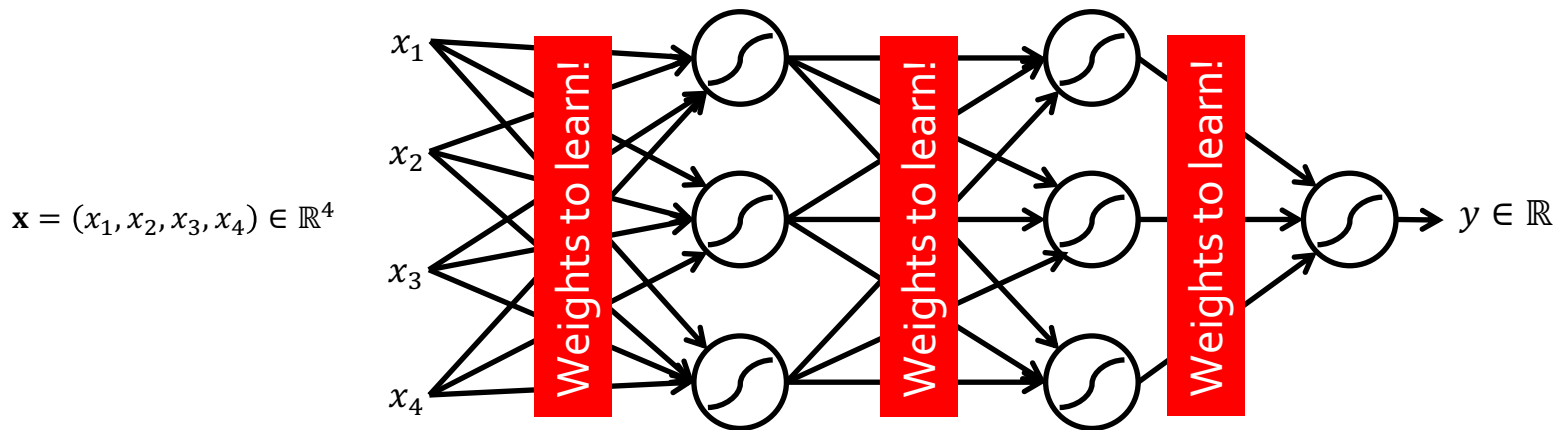


$$\mathbf{x} = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4$$



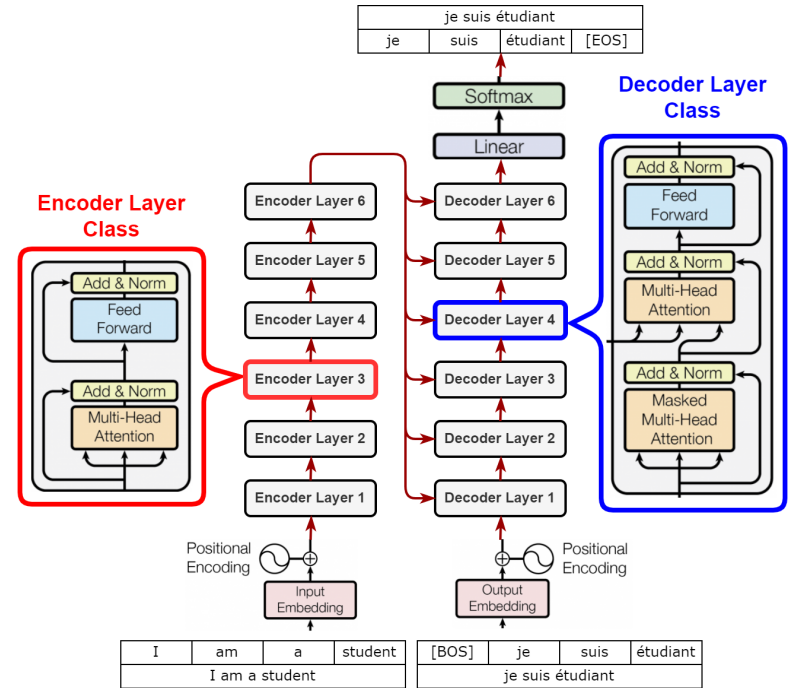
Training Neural Networks ~ Computing the Gradients

- How do you **efficiently** compute $\frac{\partial \mathcal{L}}{\partial w_i}$ for all parameters?
- It's easy to learn the final layer – it's just a linear unit.
- How about the weights in the earlier layers (i.e., before the final layer)?



Necessity of a Principled Algorithm for Gradient Computation

- **Depth** gives more representational capacity to neural networks.
- However, training **deep** nets is **not trivial**.
- Even if we have analytical formula for each gradient, they can be tedious and **must be repeated for each new architecture**.
- The solution is “Backpropagation” algorithm!



Architecture of the BERT model with over 24 layers and millions of parameters — we will study get to this model in a few weeks!

Key Intuitions Required for BP

1. Gradient Descent

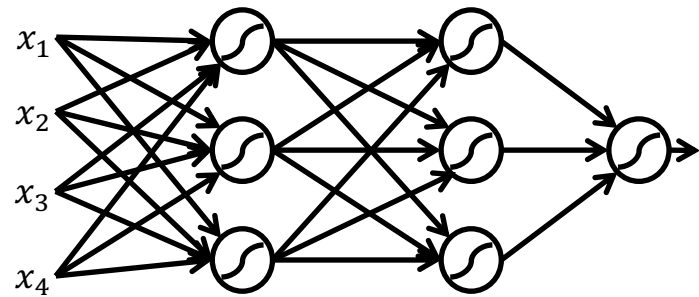
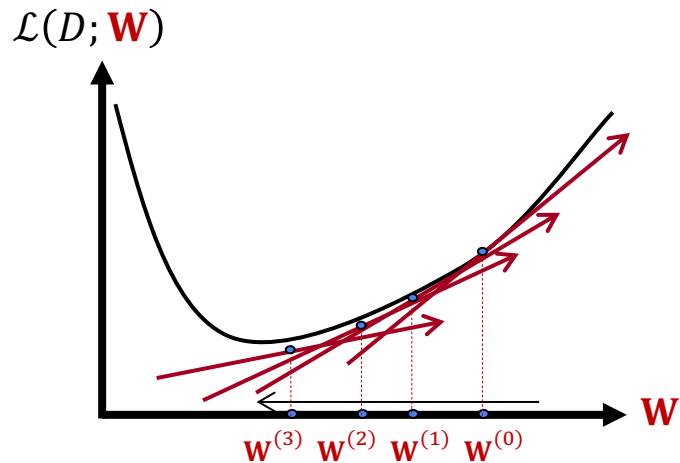
- Change the weights \mathbf{W} in the direction of gradient to minimize the error function.

2. Chain Rule

- Use the chain rule to calculate the weights of the intermediate weights

3. Dynamic Programming (Memoization)

- Memoize the weight updates to make the updates faster.



A Generic Neural Network

- Given the following definition:

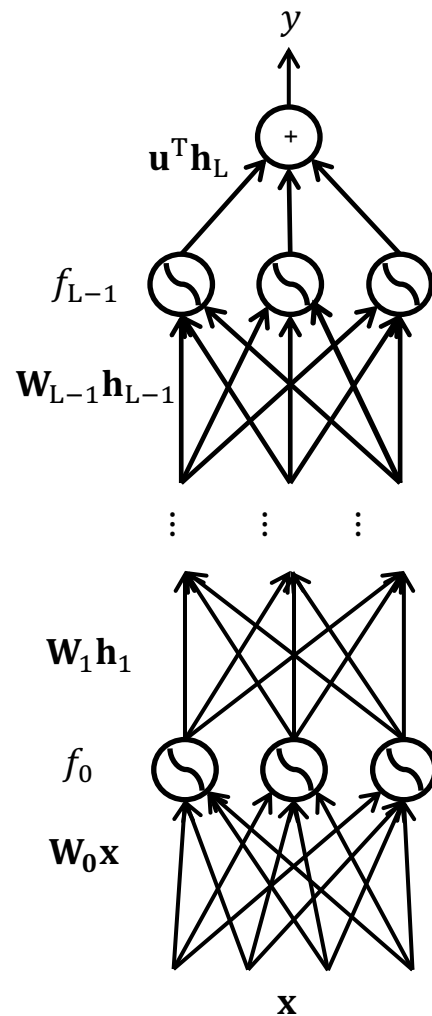
$$\mathbf{x} = \mathbf{h}_0 \in \mathbb{R}^{d_0} \text{ (input)}$$

$$\mathbf{h}_{i+1} = f_i(\mathbf{W}_i \mathbf{h}_i) \in \mathbb{R}^{d_i} \text{ (hidden layer } i, 0 \leq i \leq L - 1)$$

$$y = \mathbf{u}^T \mathbf{h}_L \in \mathbb{R} \text{ (output)}$$

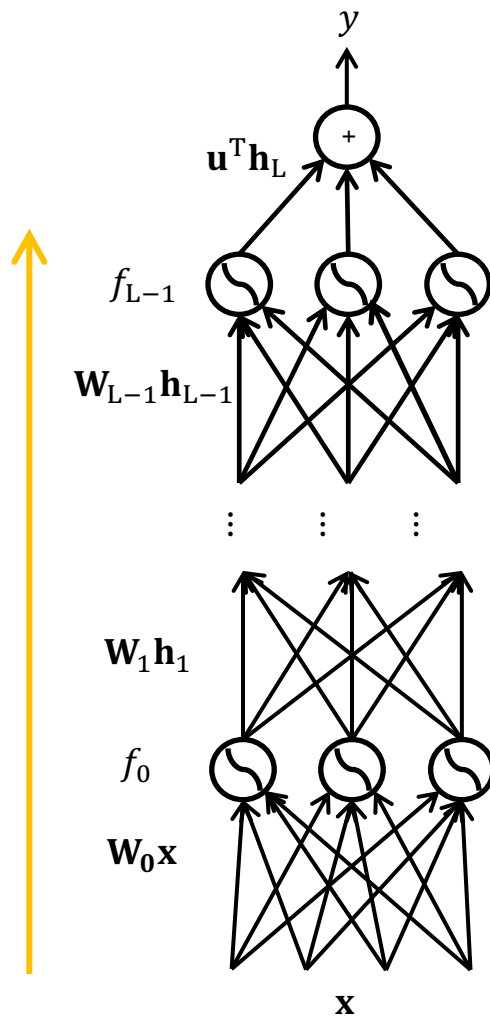
$$\mathcal{L} = \ell(y, y^*) \in \mathbb{R} \text{ (loss)}$$

- Trainable parameters: $\mathbf{W} = \{\mathbf{W}_0, \mathbf{W}_1, \dots, \mathbf{W}_L, \mathbf{u}\}$



A Generic Neural Network: Forward Step

- Given some [initial] values for the parameters, we can compute **the forward pass**, layer by layer.
- Forward pass is basically **L matrix multiplications**, each followed by an activation function.
- Matrix multiplication can be done efficiently with GPUs.
 - Therefore, **forward pass is somewhat fast**.
- Complexity of forward pass, **linear of depth $O(L)$** .



A Generic Neural Network: Direct Gradients

$$\mathbf{x} = \mathbf{h}_0 \in \mathbb{R}^{d_0} \text{ (input)}$$

$$y = \mathbf{u}^T \mathbf{h}_L \in \mathbb{R} \text{ (output)}$$

$$\mathbf{h}_{i+1} = f_i(\mathbf{W}_i \mathbf{h}_i) \in \mathbb{R}^{d_i}$$

$$\mathcal{L} = \ell(y, y^*) \in \mathbb{R} \text{ (loss)}$$

$$(0 \leq i \leq L - 1)$$

$$\mathbf{W} = \{\mathbf{W}_0, \mathbf{W}_1, \dots, \mathbf{W}_L, \mathbf{u}\}$$

We want the gradients of \mathcal{L} with respect to model parameters.

$$\bullet \nabla_{\mathcal{L}}(\mathbf{W}_{L-1}) = (\mathbf{J}_{\mathcal{L}}(\mathbf{W}_{L-1}))^T = (\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{W}_{L-1}))^T$$

$$\bullet \nabla_{\mathcal{L}}(\mathbf{W}_{L-2}) = (\mathbf{J}_{\mathcal{L}}(\mathbf{W}_{L-2}))^T = (\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}))^T$$

• ...

$$\bullet \nabla_{\mathcal{L}}(\mathbf{W}_0) = (\mathbf{J}_{\mathcal{L}}(\mathbf{W}_{L-3}))^T = (\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \dots \mathbf{J}_{\mathbf{h}_1}(\mathbf{W}_0))^T$$

3 matrix
multiplications

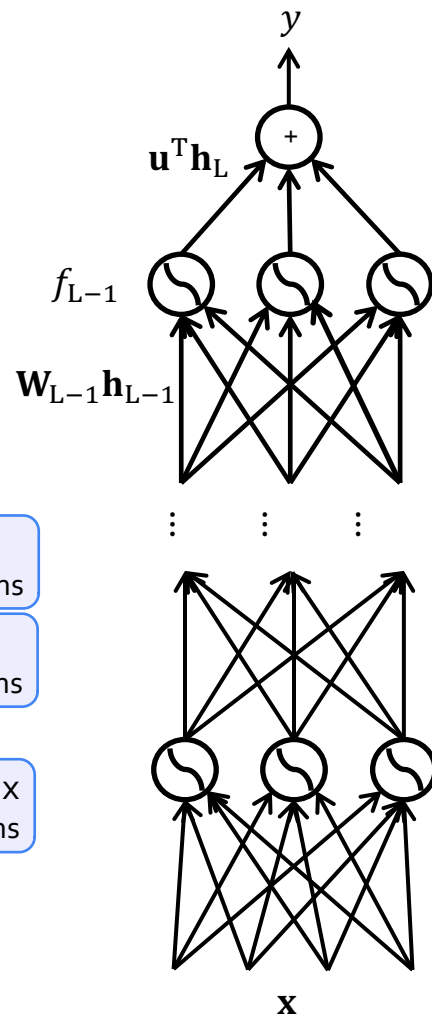
4 matrix
multiplications

$L + 2$ matrix
multiplications

In total, how many matrix multiplications are done here?

(A) $O(L)$ (B) $O(L^2)$ (C) $O(L^3)$ (D) $O(\exp(L))$

Can we do better
than this? 🤔



A Generic Neural Network: Gradients with Caching/Memoization

$$\nabla_{\mathcal{L}}(\mathbf{W}_{L-1}) = \left(\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{W}_{L-1}) \right)^T = \left(\delta_L \mathbf{J}_{\mathbf{h}_L}(\mathbf{W}_{L-1}) \right)^T$$

$$\nabla_{\mathcal{L}}(\mathbf{W}_{L-2}) = \left(\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}) \right)^T = \left(\delta_{L-1} \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}) \right)^T$$

...

$$\nabla_{\mathcal{L}}(\mathbf{W}_0) = \left(\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \dots \mathbf{J}_{\mathbf{h}_1}(\mathbf{W}_0) \right)^T = \left(\delta_1 \mathbf{J}_{\mathbf{h}_1}(\mathbf{W}_0) \right)^T$$

- Parameter gradients **depend on the gradients of the earlier layers!**
- So, when computing gradients at each layer, **we don't need to start from scratch!**
- I can **reuse gradients** computed for higher layers for lower layers (i.e., memoization).

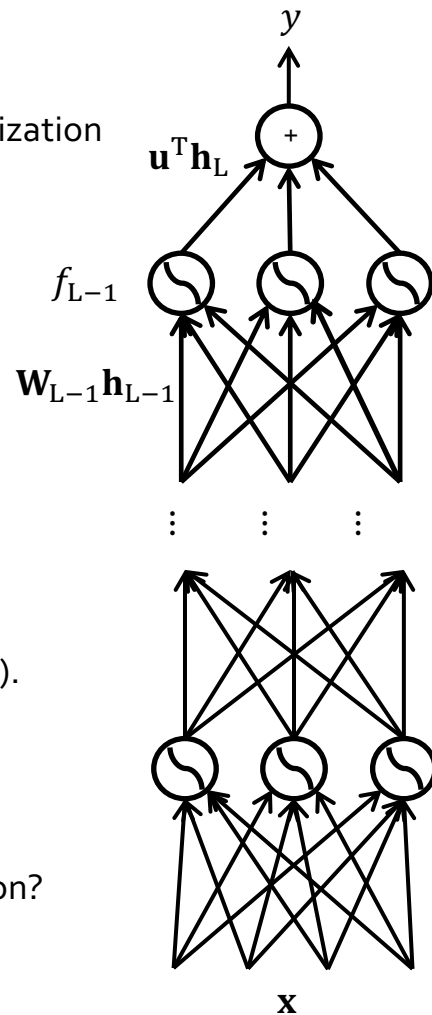
Let δ_i denote Jacobian at the output of layer i :

$$\delta_i = \mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \dots \mathbf{J}_{\mathbf{h}_i}(\mathbf{h}_{i-1})$$

$$\delta_i = \delta_{i+1} \mathbf{J}_{\mathbf{h}_i}(\mathbf{h}_{i-1})$$

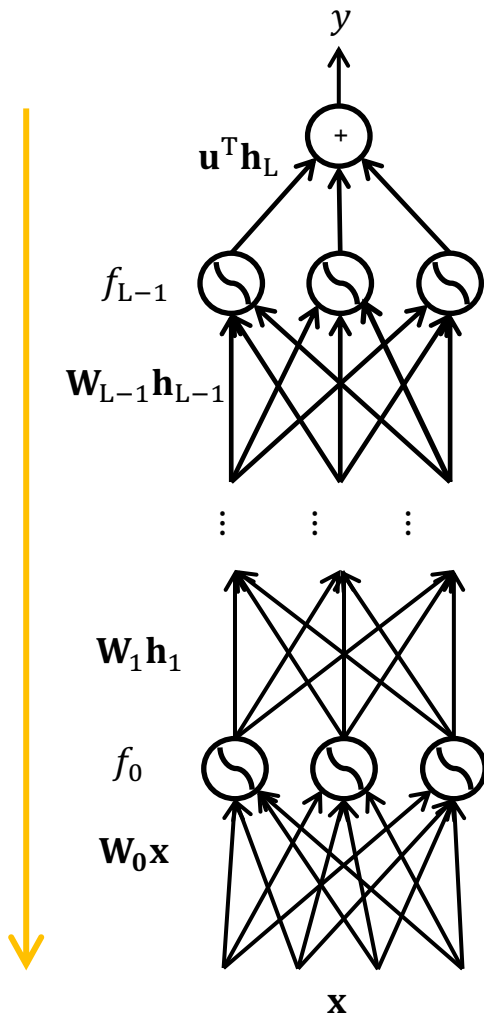
In total, how many matrix multiplications are done here when using caching/memoization?

(A) $O(L)$ (B) $O(L^2)$ (C) $O(L^3)$ (D) $O(\exp(L))$



A Generic Neural Network: Backward Step

- Backward step computes the gradients starting from the end to the beginning, layer by layer.
- Start by computing **local gradients**: $\mathbf{J}_{\mathbf{h}_i}(\mathbf{h}_{i-1})$
- Use then to compute **upstream gradients** δ_L , then δ_{L-1} , then δ_{L-2} ,
- Use these to compute **global gradients**: $\nabla_{\mathcal{L}}(\mathbf{W}_i)$
- Computational cost as a function of depth:
 - With memoization, gradient computation is a **linear** function of depth L
 - (same cost as the forward process!!)
 - Without memorization, gradients computation would grow **quadratic** with L



A Generic Neural Network: Back Propagation

Initialize network parameters with random values

Loop until convergence

Loop over training instances

i. **Forward step:**

Start from the input and compute all the layers till the end (loss \mathcal{L})

ii. **Backward step:**

Compute **local gradients**, starting from the last layer

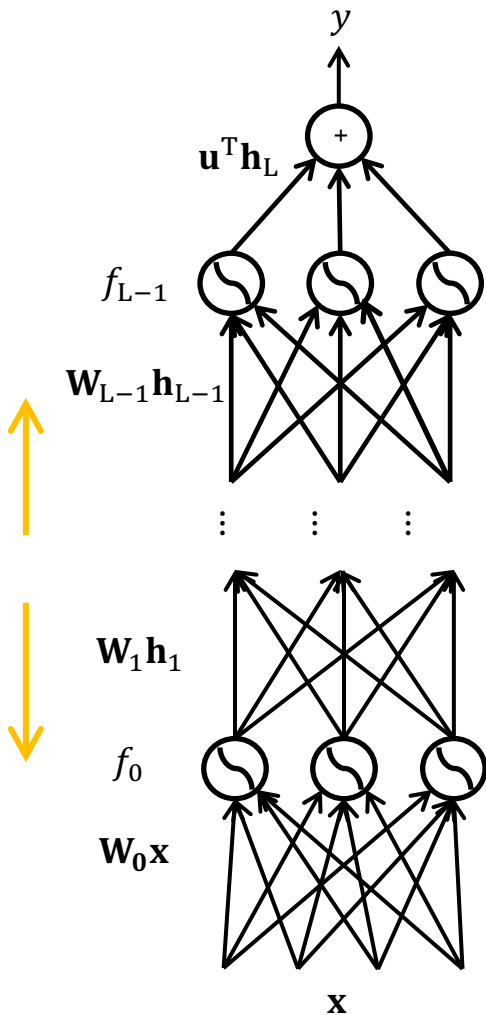
Compute **upstream gradients** δ_i values, starting from the last layer

Use δ_i values to compute global gradients $\nabla_{\mathcal{L}}(\mathbf{W}_i)$ at each layer

iii. **Gradient update:**

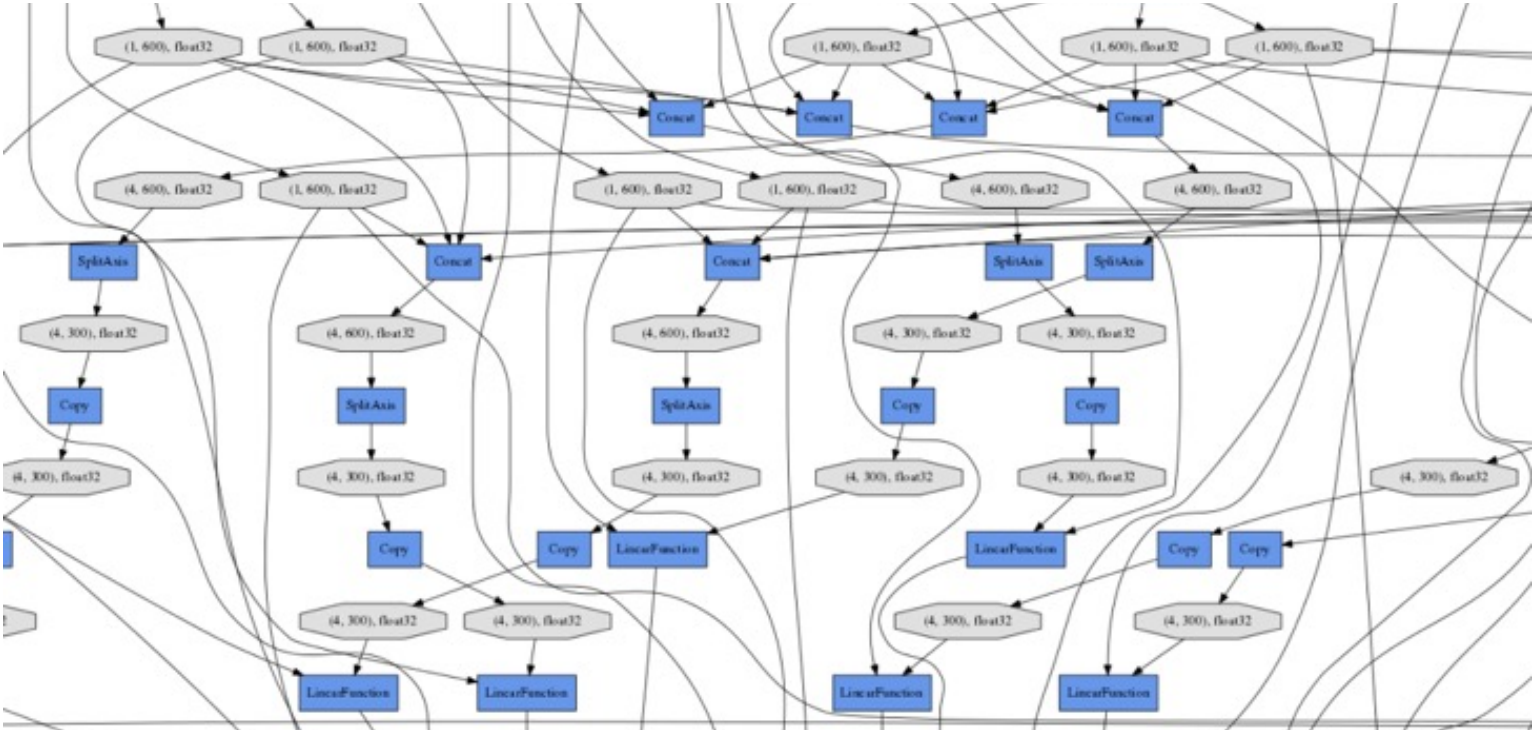
Update each parameter: $\mathbf{W}_i^{(t+1)} \leftarrow \mathbf{W}_i^{(t)} - \alpha \nabla_{\mathcal{L}}(\mathbf{W}_i)$

In practice, this step is done over **batches** of instances!



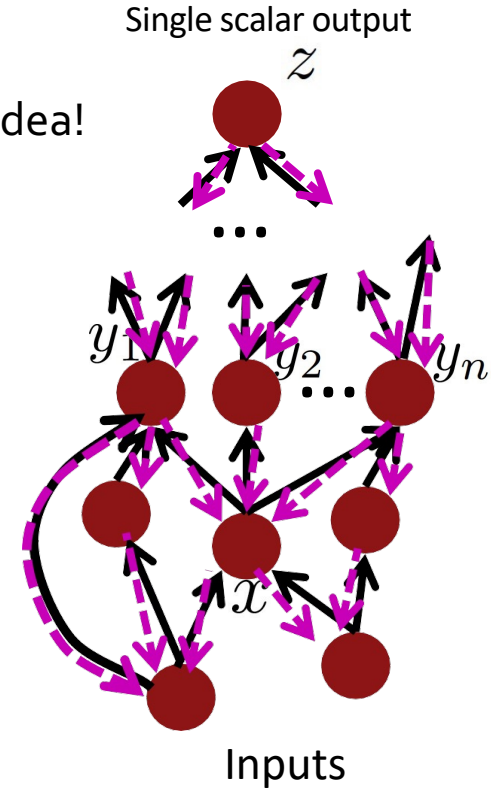
Computation Graph: Example

- In reality, networks are not as regular as the previous example ...



Back-Prop in General Computation Graph

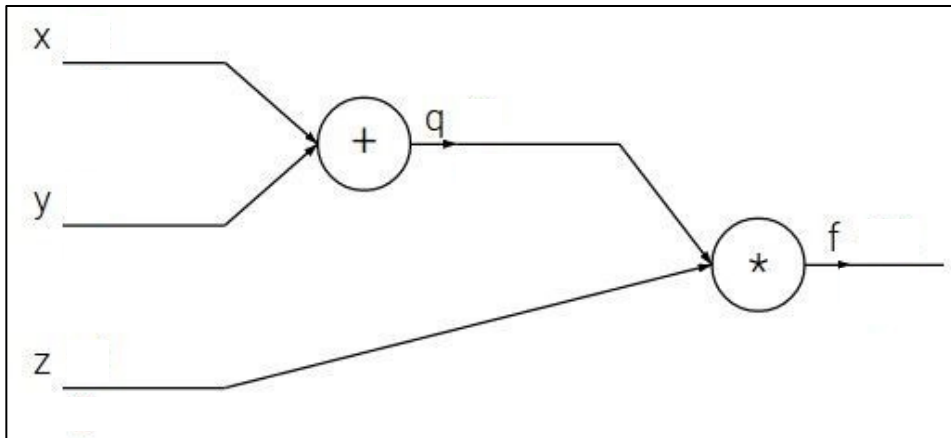
- What if the network does not have a regular structure? Same idea!
- Sort the nodes in **topological order** (what depends on what)
- Forward-Propagation:
 - Visit nodes in topological sort order and compute value of node given predecessors
- Backward-Propagation:
 - Compute **local gradients**
 - Visit nodes in reverse order and compute **global gradients** using gradients of successors



Computation Graph: An Example

$$f(x, y, z) = (x + y)z$$

- Evaluated at: $x = -2, y = 5, z = -4$

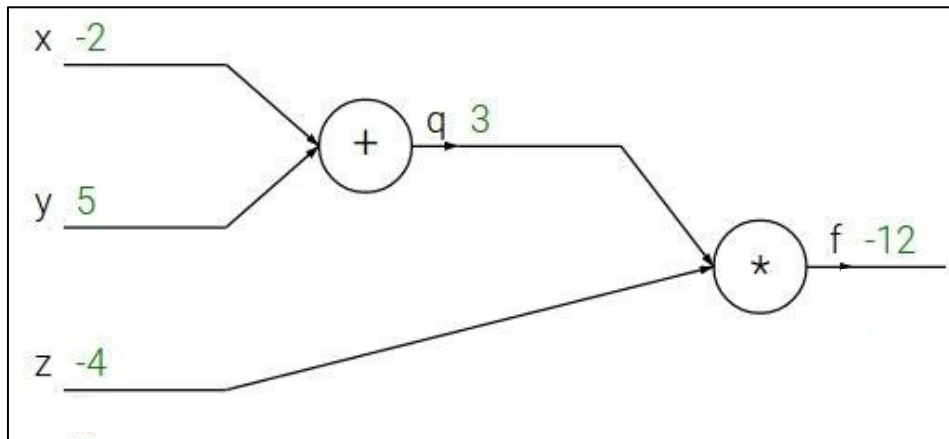


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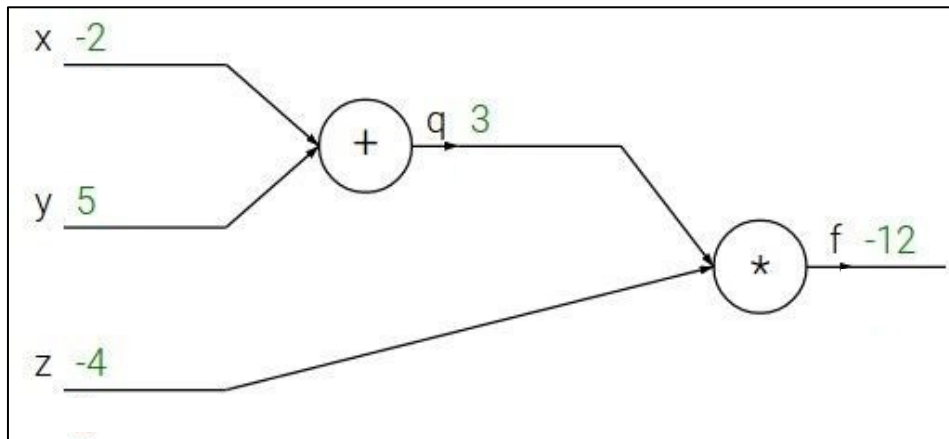
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Computation Graph: An Example

$$f(x, y, z) = (x + y)z$$

- Evaluated at: $x = -2, y = 5, z = -4$
- Start with local gradients!



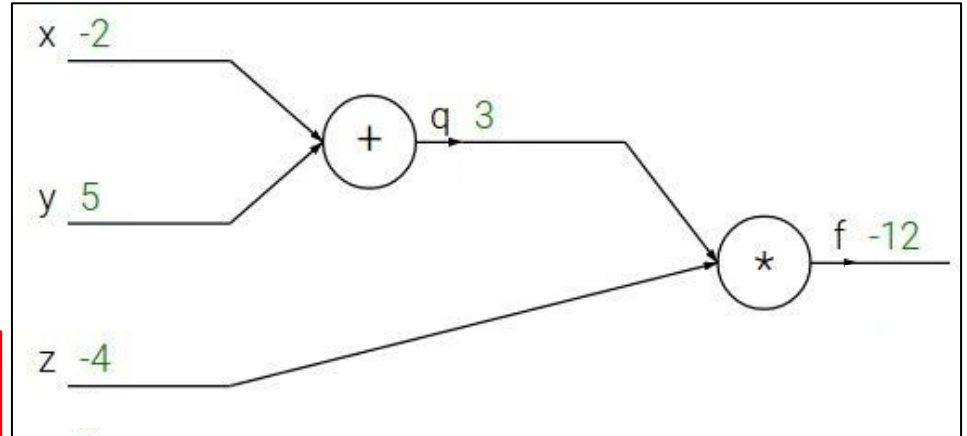
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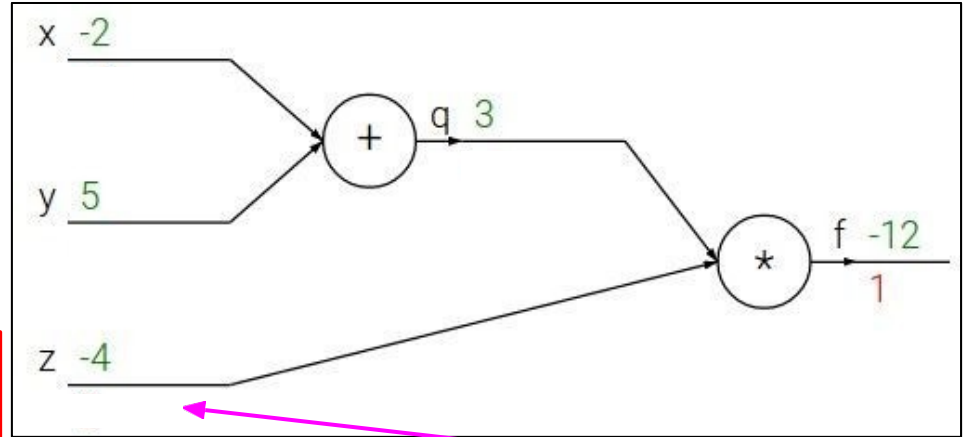
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$$\frac{\partial f}{\partial z}$$

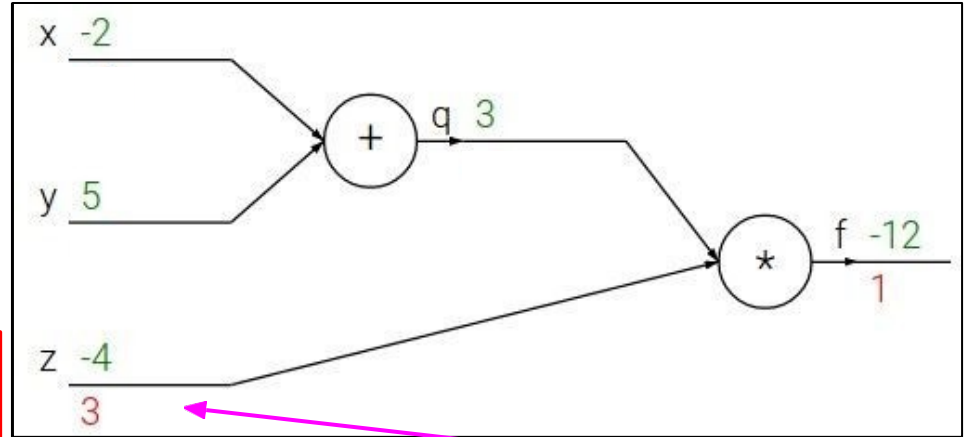
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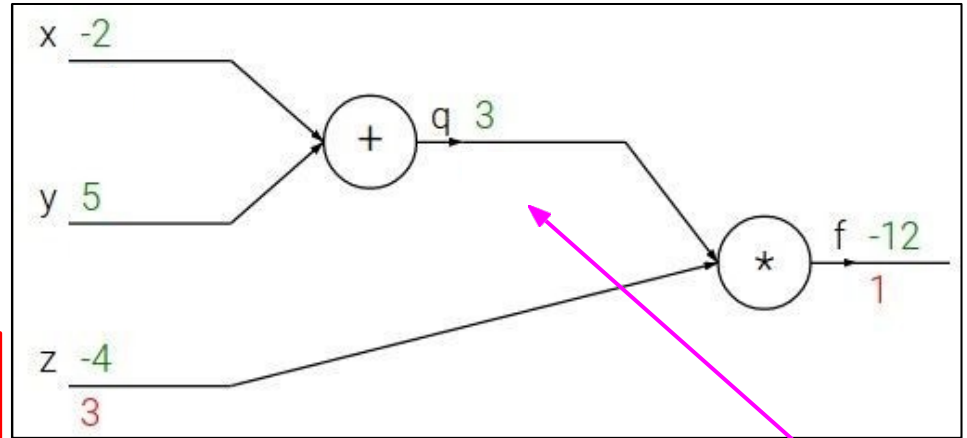
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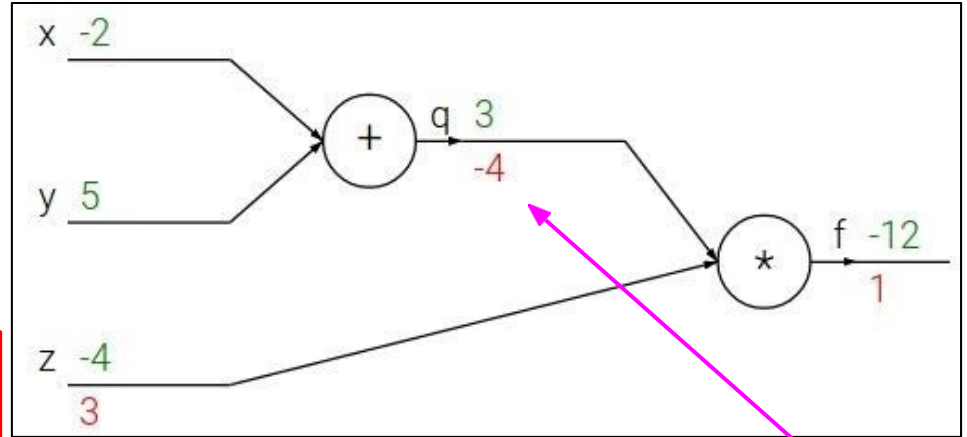
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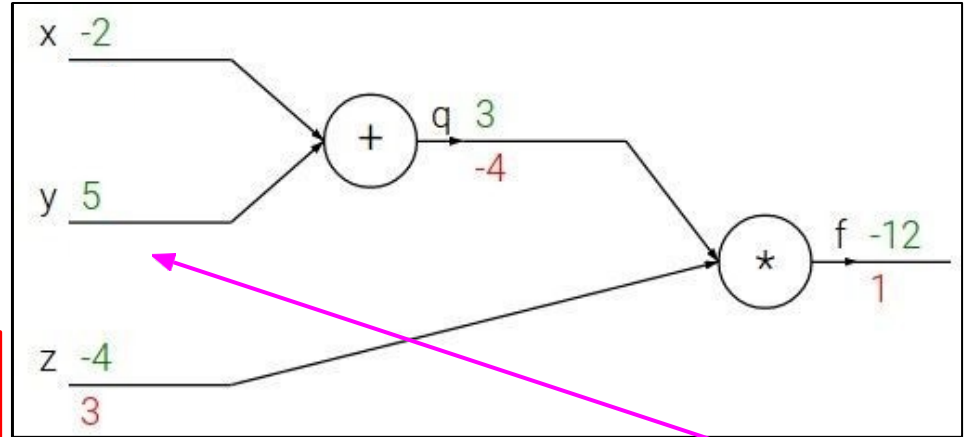
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$$\frac{\partial f}{\partial y}$$

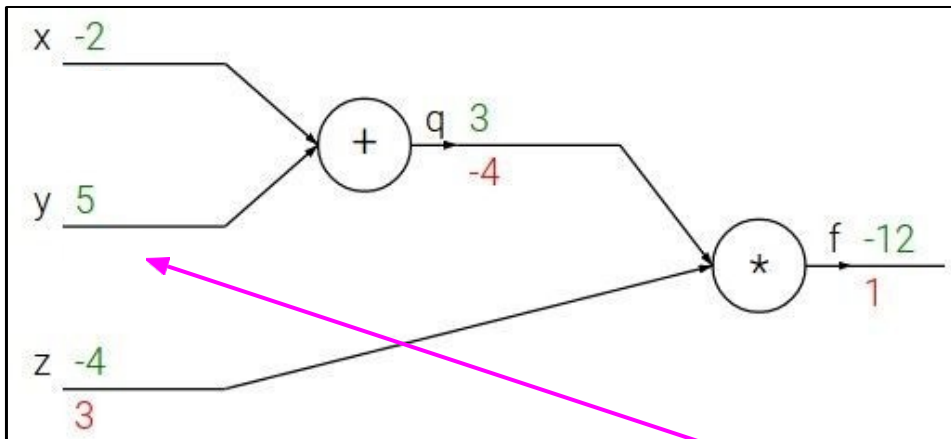
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Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

Upstream
gradient

Local
gradient

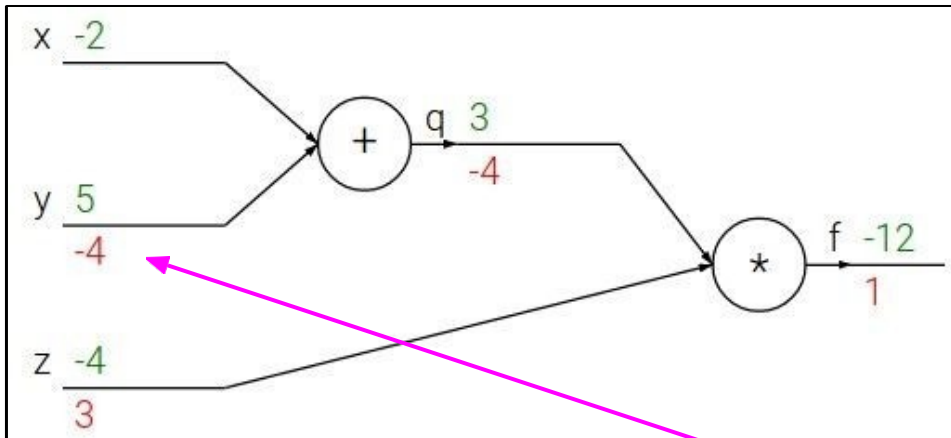
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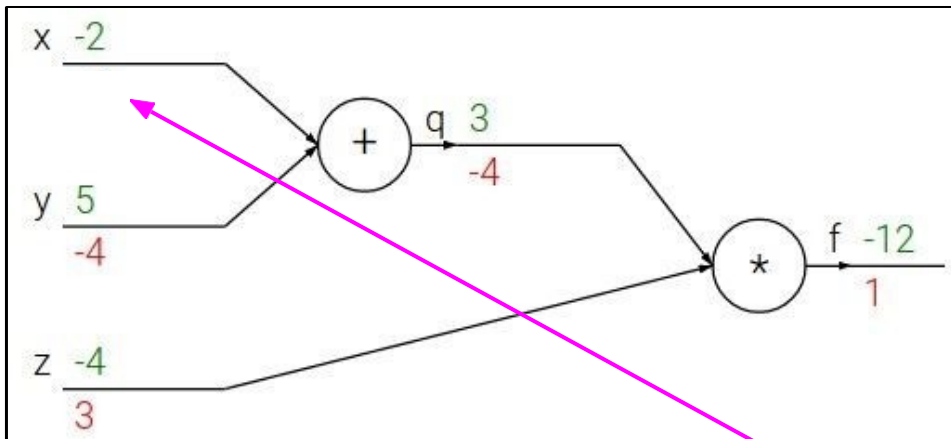
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$$\frac{\partial f}{\partial x}$$

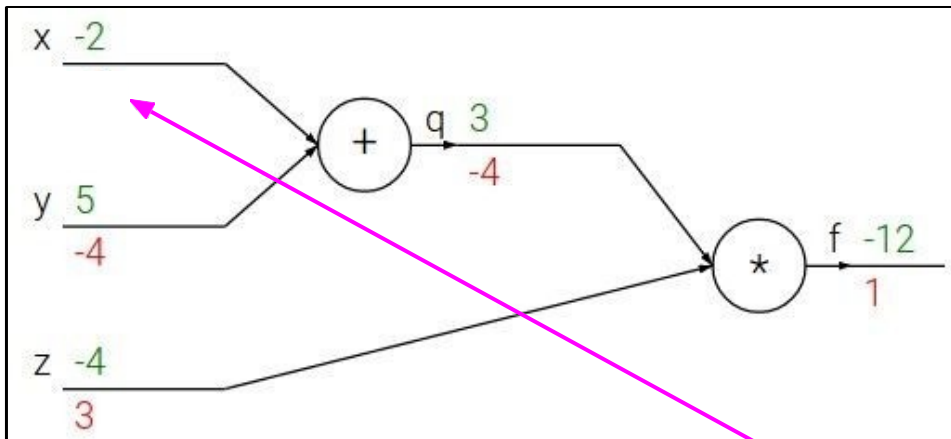
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Chain rule:

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Upstream
gradient

Local
gradient

$$\frac{\partial f}{\partial x}$$

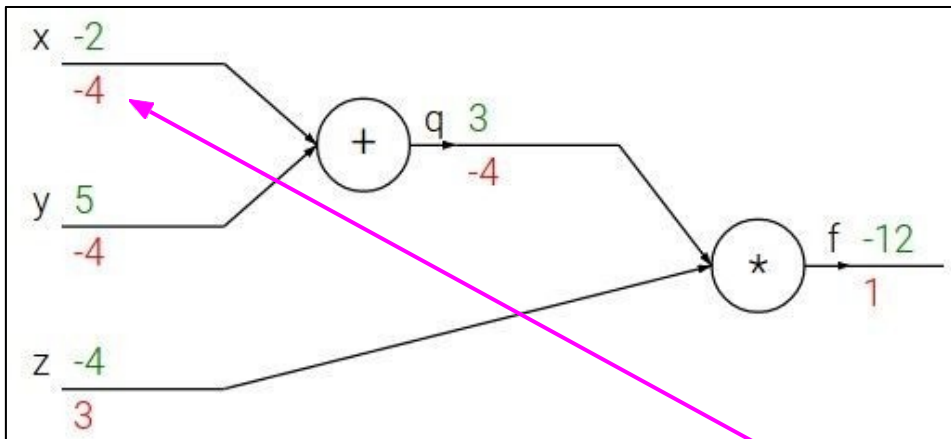
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Chain rule:

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Upstream
gradient

Local
gradient

$$\frac{\partial f}{\partial x}$$

A Generic Example

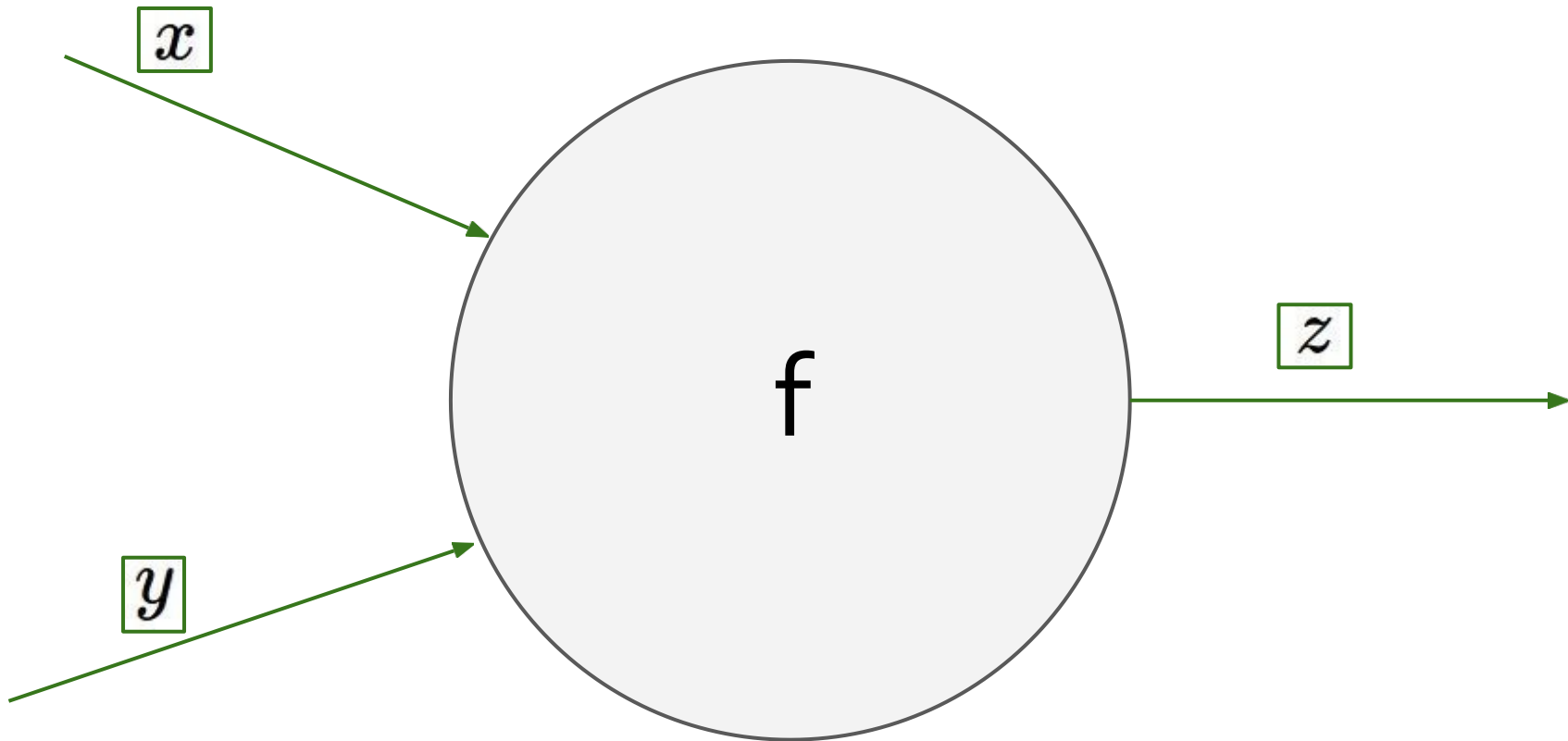


Figure from Andrej Karpathy

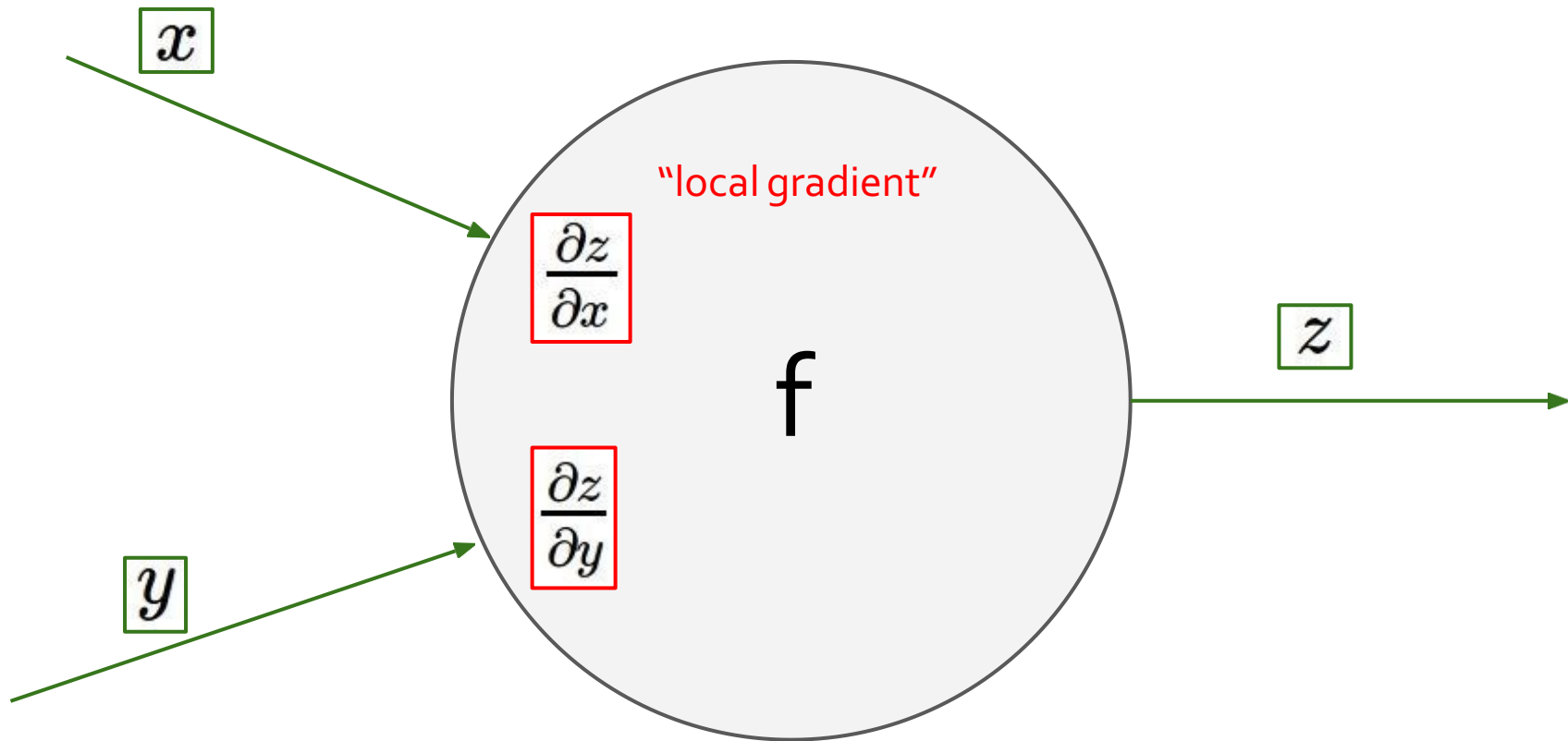


Figure from Andrej Karpathy

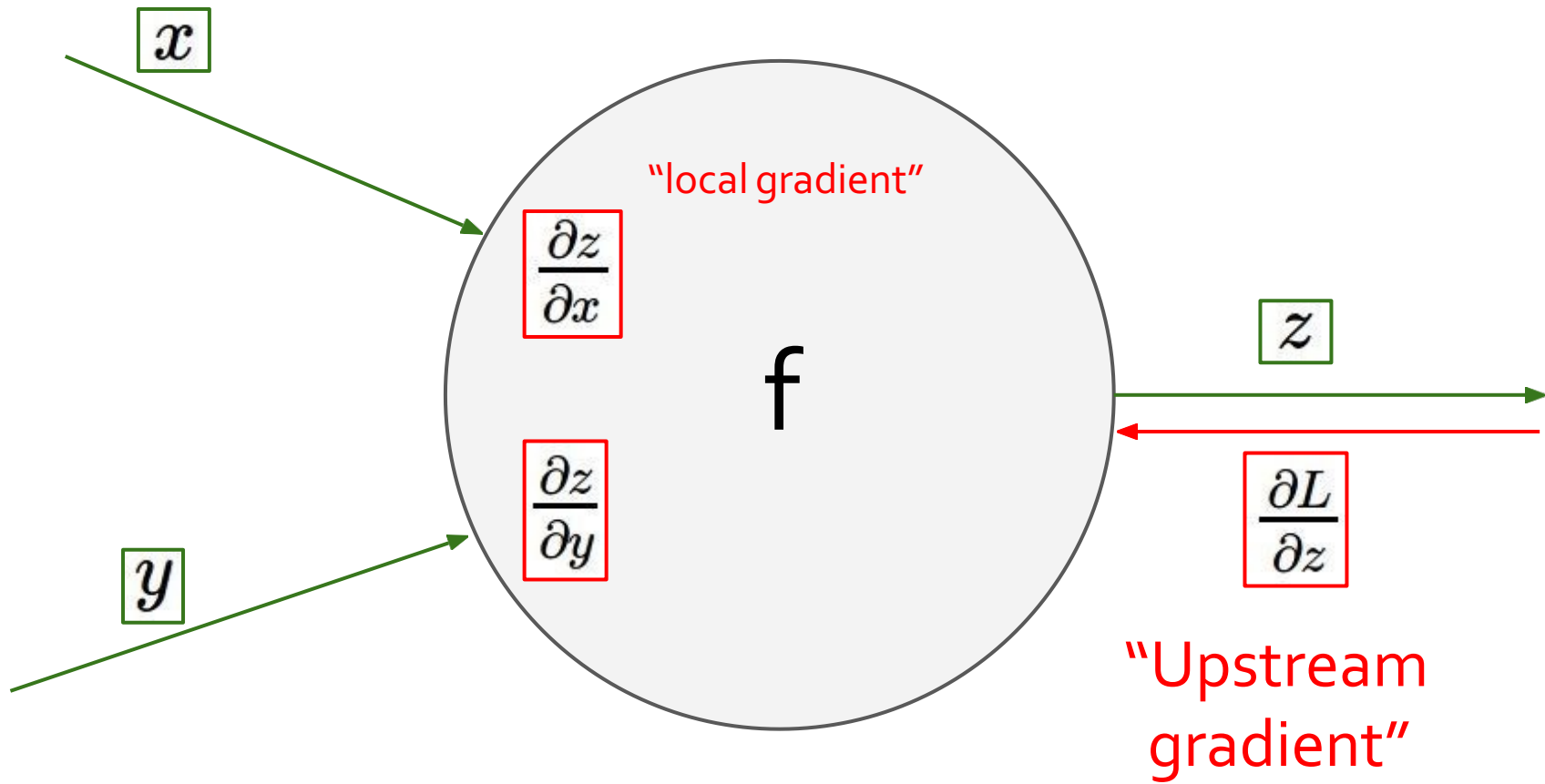
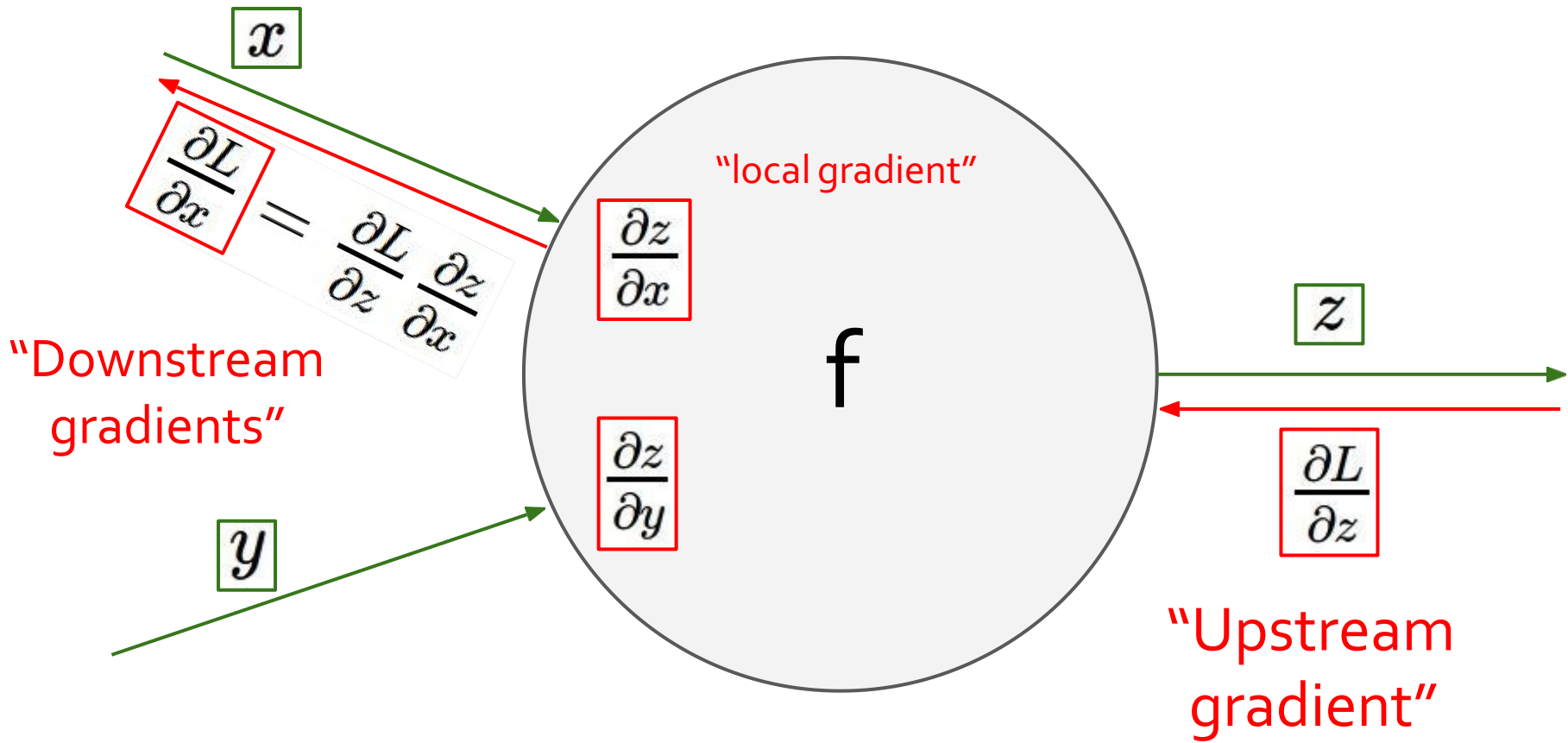
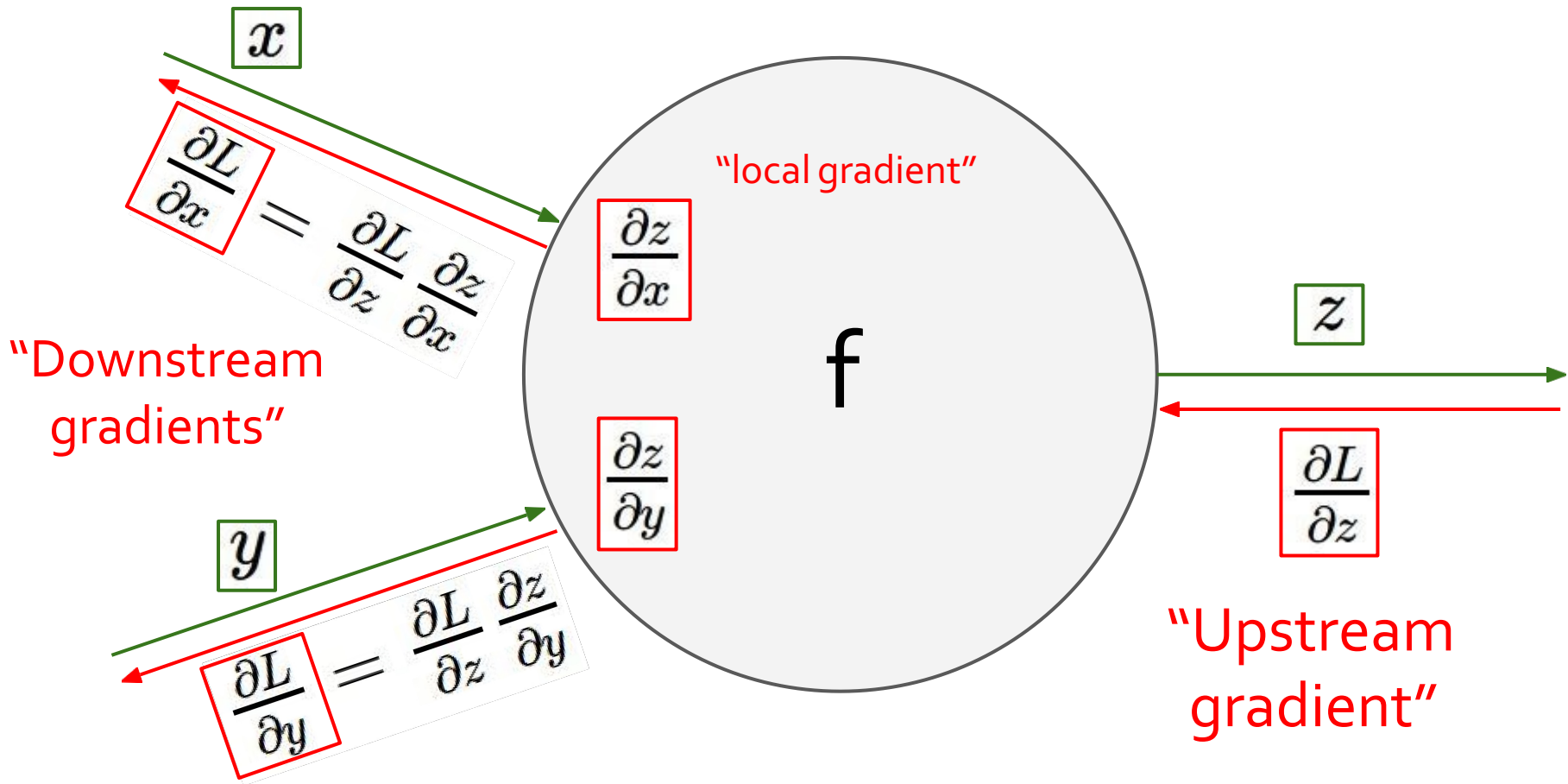
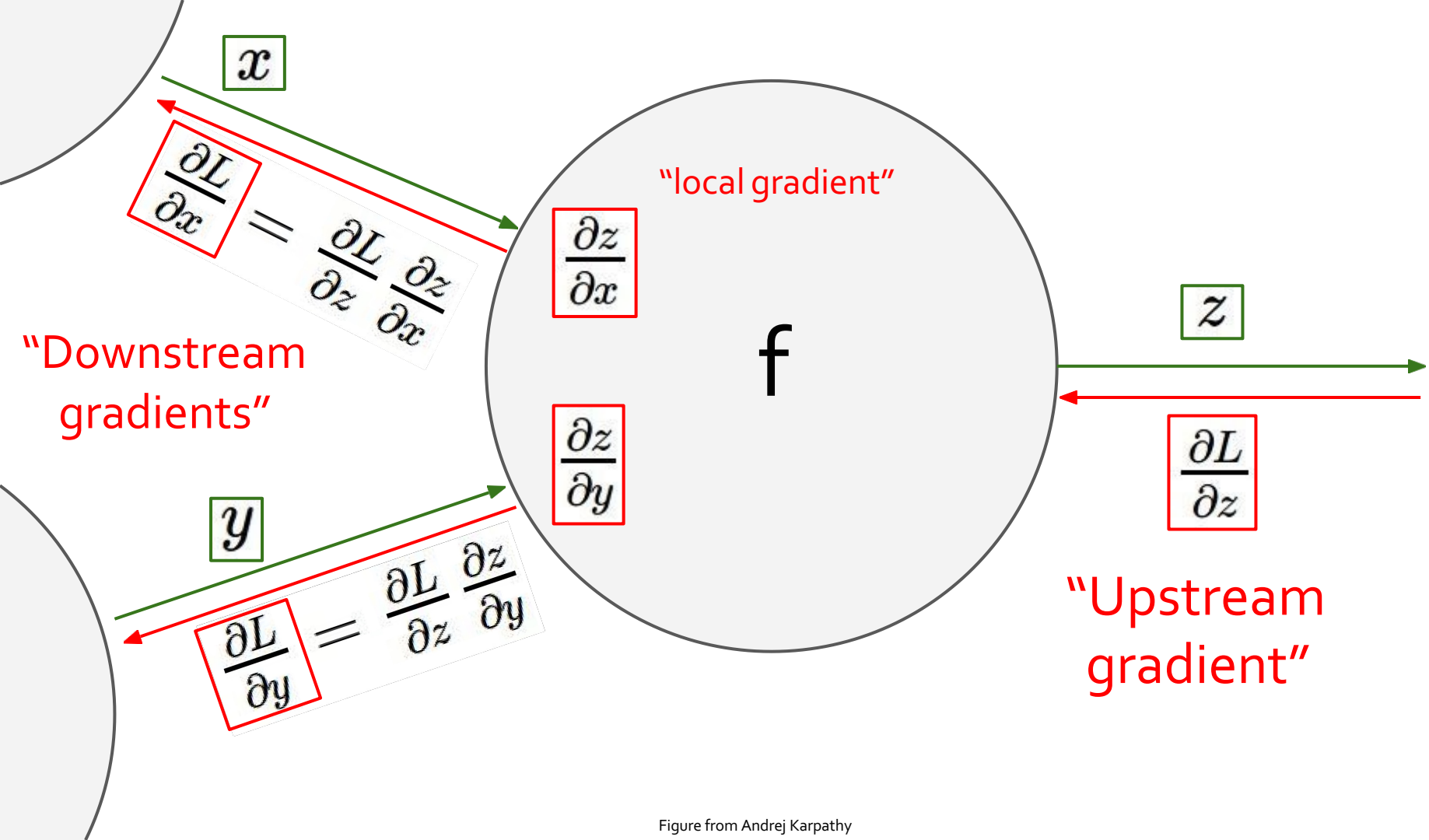


Figure from Andrej Karpathy







Demo time!

- Link: <https://playground.tensorflow.org/>

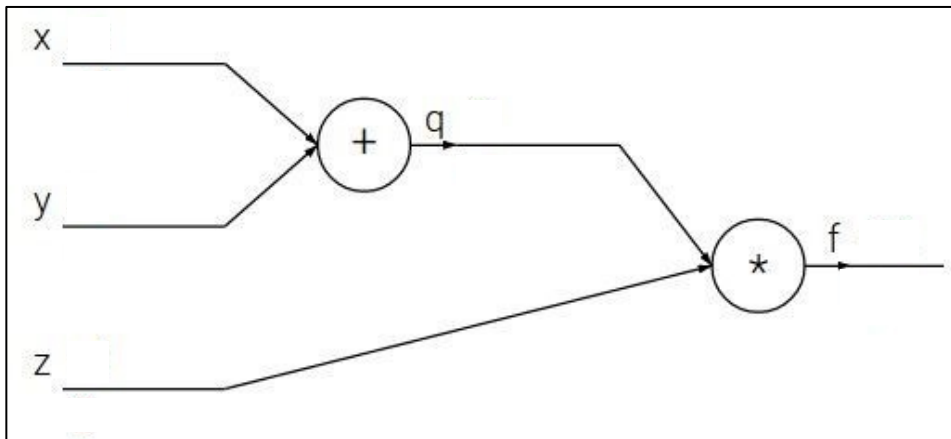
Chapter Plan

1. Feed-forward networks
2. Neural nets: brief history
3. Word2Vec as a simple neural network
4. Training neural networks: back-propagation
5. **Backprop in practice**

Backprop in PyTorch

$$f(x, y, z) = (x + y)z$$

Want: $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$



```
x = torch.tensor(-2.0, requires_grad=True)
y = torch.tensor(5.0, requires_grad=True)
z = torch.tensor(-4.0, requires_grad=True)
```

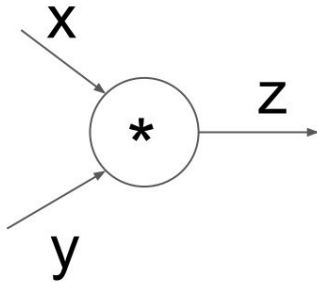
```
f = (x+y)*z # Define the computation graph
```

```
f.backward() # PyTorch's internal backward gradient computation
```

```
print('Gradients after backpropagation:', x.grad, y.grad, z.grad)
```

PyTorch's Implementation: Forward/Backward API

- PyTorch has implementation of forward/backward operations for various operators.
- Example: multiplication operator



```
class Multiply(torch.autograd.Function):  
    @staticmethod  
    def forward(ctx, x, y):  
        ctx.save_for_backward(x, y)  
        z = x * y  
        return z  
    @staticmethod  
    def backward(ctx, grad_z):  
        x, y = ctx.saved_tensors  
        grad_x = y * grad_z # dz/dx * dL/dz  
        grad_y = x * grad_z # dz/dy * dL/dz  
        return grad_x, grad_y
```

Need to cash some values for use in backward

Upstream gradient

Multiply upstream and local gradients

PyTorch Operators

PyTorch's lower-level functions translate activities to graphics processor via libraries like OpenGL

pytorch/pytorch Public

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d0a4e2e782 - pytorch / aten / src / ATen / native / vulkan / gisl / Go to file

manuelcandales and pytorchmergebot [Vulkan] Enable copying Qint8 and Qint32 tensors from cpu to vulkan. (#...	3297365 last month	History
templates	[Pytorch][Vulkan] Templatize depth wise convolution and specialize fo...	last month
adaptive_avg_pool2d.gisl	[vulkan] Add image format qualifier to gisl files (#69330)	last year
add.gisl	[Vulkan] Implement arithmetic ops where one of the arguments is a ten...	5 months ago
add_.gisl	[Vulkan] Implement arithmetic ops where one of the arguments is a ten...	5 months ago
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buffer_to_ buffer.gisl	[vulkan] Add option for buffer representations in Tensor (#87622)	2 months ago
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tanh.gisl	[vulkan] Clamp tanh activation op input to preserve numerical stabili...	10 months ago
tanh_.gisl	[vulkan] Clamp tanh activation op input to preserve numerical stabili...	10 months ago

Example Activation Functions

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SS-JIA [vulkan] Add image format qualifier to glsl files (#69330) ...

1 contributor

23 lines (17 sloc) | 710 Bytes

```
1 #version 450 core
2 #define PRECISION $precision
3 #define FORMAT $format
4
5 layout(std430) buffer;
6
7 /* Qualifiers: layout - storage - precision - memory */
8
9 layout(set = 0, binding = 0, FORMAT) uniform PRECISION restrict writeonly image3D uO
10 layout(set = 0, binding = 1) uniform PRECISION sampler3D uI
11 layout(set = 0, binding = 2) uniform PRECISION restrict Block {
12     ivec4 size;
13 } uBlock;
14
15 layout(local_size_x_id = 0, local_size_y_id = 1, local_size_z_id = 2) in;
16
17 void main() {
18     const ivec3 pos = ivec3(gl_GlobalInvocationID);
19
20     if (all(lessThan(pos, uBlock.size.xyz)) {
21         imageStore(uOutput, pos, 1/(1+exp(-1*texelFetch(uInput, pos, 0))));
22     }
```

master pytorch / aten / src / ATen / native / vulkan / glsl / tanh.gsl

SS-JIA [vulkan] Clamp tanh activation op input to preserve numerical stabili... ...

2 contributors

27 lines (21 sloc) | 777 Bytes

```
1 #version 450 core
2 #define PRECISION $precision
3 #define FORMAT $format
4
5 layout(std430) buffer;
6
7 /* Qualifiers: layout - storage - precision - memory */
8
9 layout(set = 0, binding = 0, FORMAT) uniform PRECISION restrict writeonly image3D uOutput;
10 layout(set = 0, binding = 1) uniform PRECISION sampler3D uInput;
11 layout(set = 0, binding = 2) uniform PRECISION restrict Block {
12     ivec4 size;
13 } uBlock;
14
15 layout(local_size_x_id = 0, local_size_y_id = 1, local_size_z_id = 2) in;
16
17 void main() {
18     const ivec3 pos = ivec3(gl_GlobalInvocationID);
19
20     if (all(lessThan(pos, uBlock.size.xyz)) {
21         const vec4 intex = texelFetch(uInput, pos, 0);
22         imageStore(
23             uOutput,
24             pos,
25             tanh(clamp(intex, -15.0, 15.0)));
26     }
```

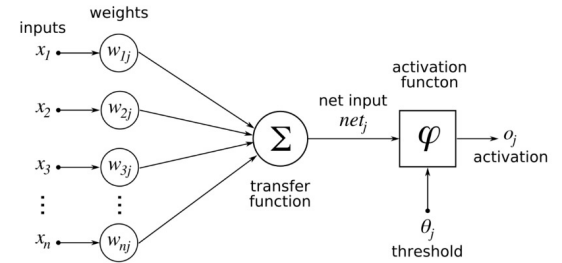
Why Learn All These Details About Backprop?

- **Modern deep learning frameworks compute gradients for you!**
- But why take a class on compilers or systems when they are implemented for you?
 - Understanding what is going on under the hood is useful!
- Backpropagation doesn't always work perfectly out of the box
 - Understanding why is crucial for debugging and improving models

Backprop in Practice

Activation Functions

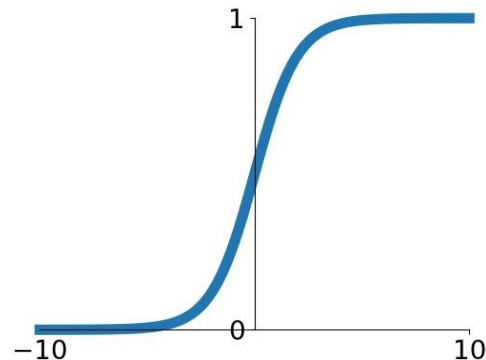
- How do you choose what activation function to use?
- In general, it is problem-specific and might require trial-and-error.
- Here are some tips about popular action functions.



Activation Functions : Sigmoid



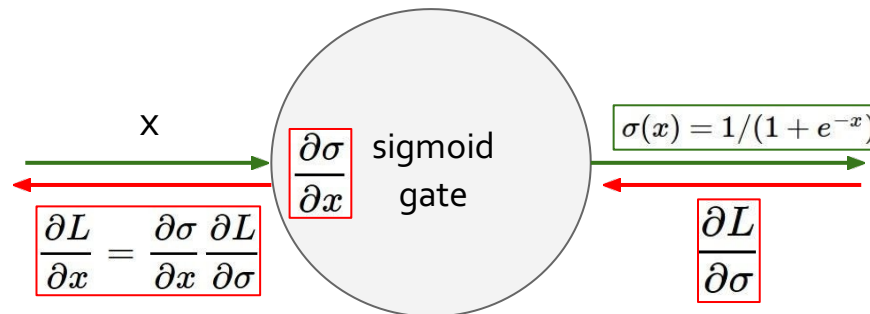
- Squashes numbers to range $[0,1]$
- Historically popular, **interpretation as “firing rate” of a neuron**
- **Key limitation:** Saturated neurons “kill” the gradients
- Whenever $|x| > 5$, the gradients are basically zero.



$$\sigma(x) = 1/(1 + e^{-x})$$

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

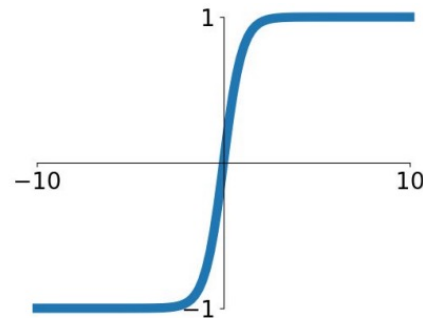
If all the gradients flowing back will be zero and weights will never change.



Activation Functions : Tanh



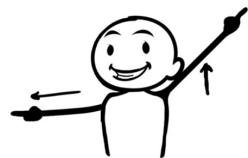
- Symmetric around $[-1, 1]$
- Still saturates $|x| > 3$ and “kill” the gradients
- Zero-centered — good for stacking hidden layers



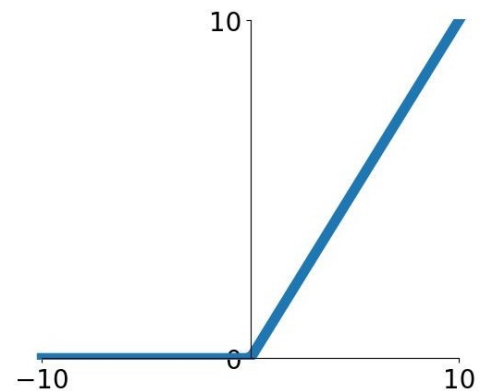
$\tanh(x)$

[LeCun et al., 1991]

Activation Functions : ReLU



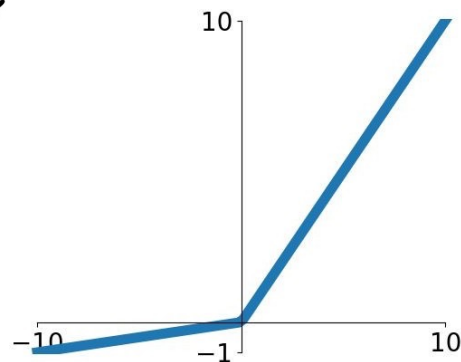
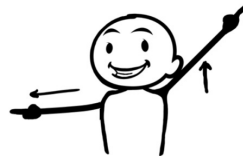
- Computationally efficient
- In practice, converges faster than sigmoid/tanh in practice
- Does not saturate (in +region) — will die less!



ReLU
(Rectified Linear Unit)

[Krizhevsky et al., 2012]

Activation Functions : Leaky ReLU



$$f(x) = \max(0.01x, x)$$

- Does not saturate — will not die.
- Computationally efficient
- In practice it converges faster than sigmoid/tanh in practice
- Other parametrized variants:
 - Parametric Rectifier (PReLU): $f(x) = \max(\alpha x, x)$ [He et al., 2015]
 - Maxout: $\max(w_1^T x + b_1, w_2^T x + b_2)$ [Goodfellow et al., 2013]
- Provide more flexibility, though at the cost of more learnable parameters.
 - For example, Maxout doubles the number of parameters.

How do You Choose What Activation Function to Use?

- In general, it is problem-specific and might require trial-and-error.
- A useful recipe:
 1. Generally, ReLU is a good activation to start with.
 2. Time/compute permitting, you can try other activations to squeeze out more performance.

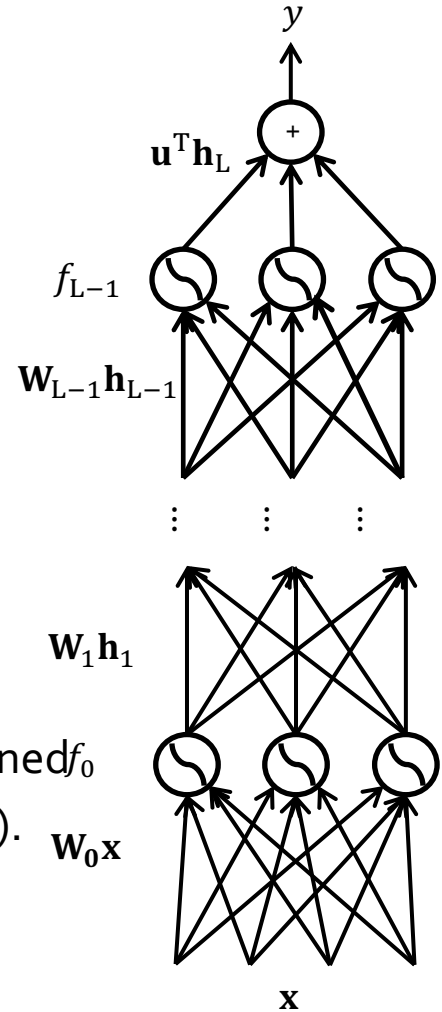
Exploding/Vanishing Gradients

- Remember gradient computation at layer $L - k$:

$$\nabla_{\mathcal{L}}(\mathbf{W}_{L-k}) = \underbrace{\left(\mathbf{J}_{\ell}(y) \mathbf{J}_y(\mathbf{h}_L) \mathbf{J}_{\mathbf{h}_L}(\mathbf{h}_{L-1}) \mathbf{J}_{\mathbf{h}_{L-1}}(\mathbf{W}_{L-2}) \dots \mathbf{J}_{\mathbf{h}_{L-k+1}}(\mathbf{W}_{L-k}) \right)^T}_{O(k)\text{-many matrix multiplication}}$$

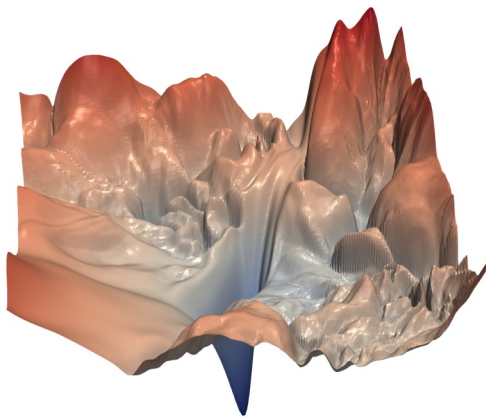
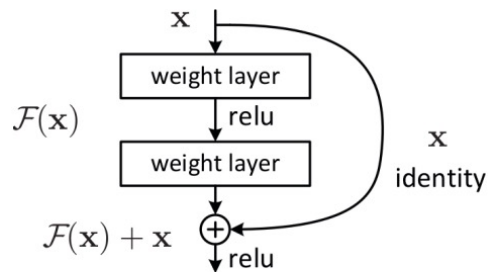
$O(k)$ -many matrix multiplication

- This matrix multiplication could quickly approach
 - ∞ , if the matrix elements are a large — exploding gradients.
 - 0, if the matrix elements are small — vanishing gradients.
- For those interested, convergences of matrix powers is determined by its largest eigenvalue (out of scope for this class, extra credit).
- $\infty/0$ gradients would kill learning (no flow of information).

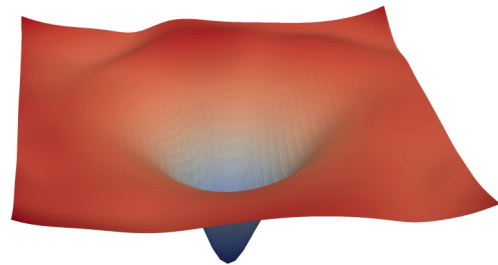


Residual Connections/Blocks

- Create direct “information highways” between layers.
- Shown to diminish the effect of vanishing/exploding gradients
 - Early in the training, there are fewer layers to propagate through.
 - The network would restore the skipped layers, as it learns richer features.
 - It is also shown to make the optimization objective smoother.
 - Fun fact: [the paper](#) introducing residual layers (He et al. 2015) is the most cited paper of century.



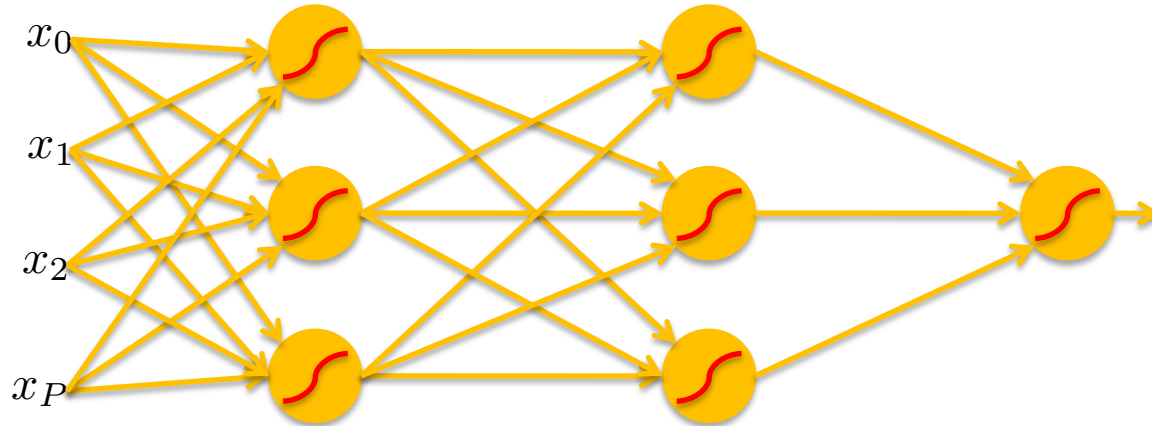
(a) without skip connections



(b) with skip connections

Weight Initialization

- Initializing all weights with a fixed constant (e.g., 0) is a very bad idea! (why?)



- If the neurons start with the same weights, then all the neurons will follow the same gradient, and will always end up doing the same thing as one another.

Weight Initialization

- Better idea: initialize weights with random Gaussian noise.

```
x = torch.tensor.empty(3, 5)
nn.init.normal_(w)
```

- There are fancier initializations (Xavier, Kaiming, etc.) that we won't get into.

Comments on Training

- **No guarantee of convergence**; neural networks form non-convex functions with multiple local minima
- In practice, many large networks can be trained on large amounts of data for realistic problems.
- May be hard to set learning rate and to select number of hidden units and layers.
- **Many steps** (tens of thousands) may be needed for adequate training. Large data sets may require many hours of CPU
- **Termination criteria**: Number of epochs; Increased error on a validation set.
- To **avoid local minima**: several trials with different random initial weights with majority or voting techniques

Over-training Prevention

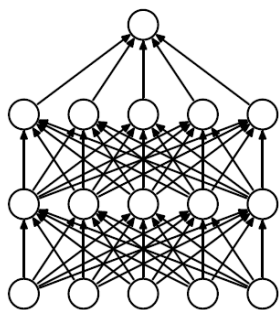
- Running too many epochs and/or a NN with many hidden layers may lead to an **overfit** network
- Keep a **held-out validation** set and evaluate accuracy after every epoch
- Early stopping: maintain weights for best performing network on the validation set and return it when performance decreases significantly beyond that.
- To avoid losing training data to validation:
 - Use 10-fold cross-validation to determine the average number of epochs that optimizes validation performance
 - Train on the full data set using this many epochs to produce the final results

Over-fitting prevention

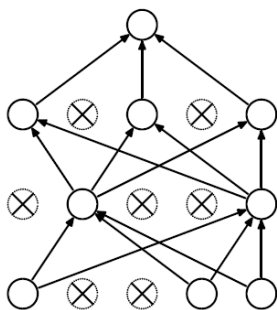
- **Too few hidden units** prevent the system from adequately fitting the data and learning the concept.
- Using **too many hidden units** leads to over-fitting.
- Similar cross-validation method can be used to determine an appropriate number of hidden units. (general)
- Another approach to prevent over-fitting is weight-decay: all weights are multiplied by some fraction in $(0,1)$ after every epoch.
 - Encourages smaller weights and less complex hypothesis
 - Equivalently: change Error function to include a term for the sum of the squares of the weights in the network. (general)

Dropout training

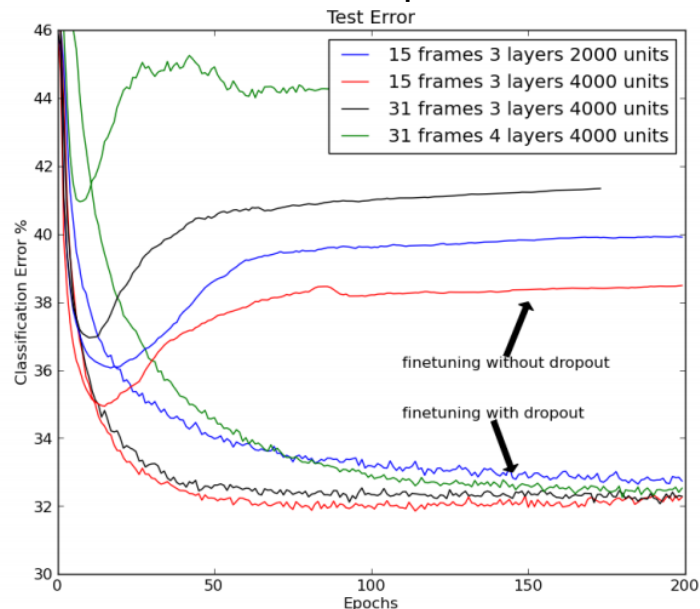
- In each forward pass, randomly set some neurons to zero
- Probability of dropping is a hyperparameter; 0.5 is common
- Dropout is implicitly an ensemble (average) of models that share parameters.
 - Each binary mask is one model
 - For example, an FC layer with 4096 units has $2^{4096} \sim 10^{1233}$ possible masks!
 - Only $\sim 10^{82}$ atoms in the universe ...



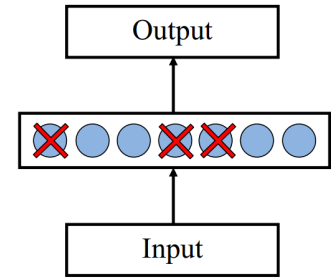
(a) Standard Neural Net



(b) After applying dropout.



Dropout During Test Time



- The issue for test time is that Dropout adds randomization.
 - Each dropout mask would lead to a slightly different outcome.
- In ideal world, we would like to “average out” the outcome across all the possible random masks:

- Not feasible.
$$y = f(x) = E_z[f(x, z)] = \int p(z)f(x, z)dz$$

- The alternative is to not apply dropout. Without dropout, the input values to each neuron would be higher than what was seen during the training (mismatch between train/test).

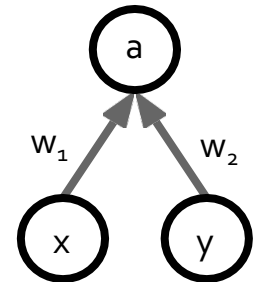
- **Example:** Input to activation during:

- training time:
$$E[a] = \frac{1}{4}(w_1x_1 + w_2x_2) + \frac{1}{4}(0 + 0) + \frac{1}{4}(0 + w_2x_2) + \frac{1}{4}(w_1x_1 + 0) = \frac{1}{2}(w_1x_1 + w_2x_2)$$

- test time:
$$E[a] = w_1x_1 + w_2x_2$$

- **Solution:** **scale the values** proportional to dropout probability.

- Can be applied in either testing (scaling down) or training (scaling up).



Dropout in Practice

Just call the PyTorch function!

```
dropout = nn.Dropout(p=0.2)
x = torch.randn(20, 16)
y = dropout(x)
```

It automatically

- activates the dropout for **training**.
- deactivates it during **evaluations** and scales the values according to its parameter.

```
# training step
...
model.train()
...
```

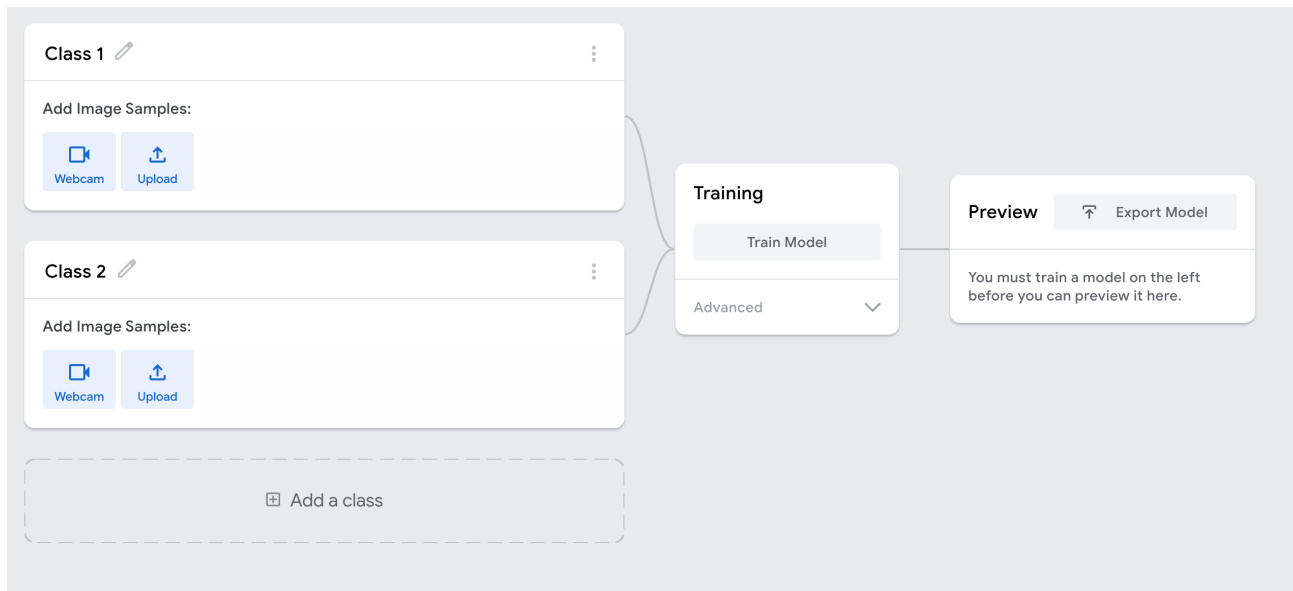
```
# evaluate model:
...
model.eval()
...
```

The Only Time You Want to Overfit: The First Tryout

- A model with buggy implementation (e.g., incorrect gradient calculations or updates) cannot learn anything.
- Therefore, a good and easy sanity check is to see if you can overfit few examples.
 - This is really the first test you should do, before any hyperparameter tuning.
- Try to train to 100% training accuracy/performance on a small sample (<30) of training data and monitor the **training** loss trends.
 - Does it down? If not, something must be wrong.
 - Try checking the **learning rate** or modifying the initialization.
 - If those don't help, check the gradients.
 - If they're **NaN** or **Inf**, might indicate **exploding gradients**.
 - If they're **zeros**, might indicate **vanishing gradients**.

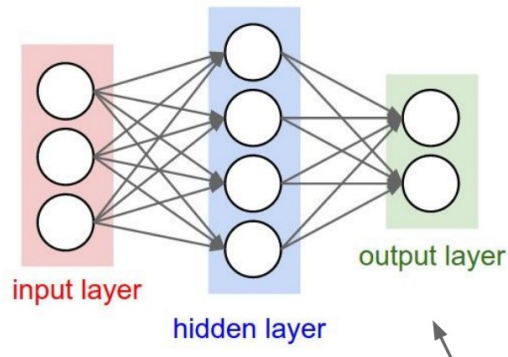
Demo Time!

- <https://teachablemachine.withgoogle.com/>



Chapter Summary

- Feed-forward network architecture
- Word2Vec is just a feedforward net!
 - And we can easily extend it!



- We learned Back-Prop, the most important algorithm in neural networks! 🎉
 - Recursively (and hence efficiently) apply the chain rule along computation graph
- Lots of empirical tricks for training neural networks:
 - First test: check if you can overfit.
 - Dropout
 - Be mindful of activations
 - Careful of exploding/vanishing gradients